Part 2

Quantum Mechanics:

Concepts and Applications

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Part 1 of this four part series reviewed the history, development, and interpretation of quantum mechanics. This was done in a nonmathematical fashion appropriate to a general background of the field.

Part 2 reviewed some of the details of quantum theoretical methods. The objective was to lay out the gist of the field with a minimal level of mathematics.

Part 3 reviews issues in Classical and Quantum Information Theory, focusing on Cryptology and Computing

Part 4 is a Technical Appendix

Revision 1: Addition of section "The Atom"
Addition of Appendix on Complex Numbers

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Contents

Background Concepts

Quantum Systems

A *quantum system* is any piece of the quantum world considered in isolation. The simplest quantum system is a one-particle system, for example an electron or a photon. The underlying assumption that this single particle acts in isolation is, of course, not "realistic": the electron is in a cloud of other electrons, neutrons, protons and other particles. To paraphrase John Donne, "No electron is an island, entire unto itself." But the assumption is useful because it allows one to describe the simplest level of quantum mechanics.

The next most tractable quantum system is a two-particle system, as when a photon decays into an electron and an antielectron (positron). Again, the assumption of isolation is not realistic, but it is useful. Even this simple system can be complicated because in two particles can be *entangled*, a characteristic described in Part 1 that will be more fully developed later.

Dirac Notation and Linear Algebra of Quantum States

A quantum state is described by one or more numbers representing the values of the characteristics of the state. Suppose that there are n characteristics describing a quantum state, call them $s_1, s_2,\ldots,s_n$. Those state characteristics might be positions in space and time: t for time, z for position in the north-south direction, x for position in the east-west direction, and y for position in the in-out direction (pointing toward or away from the reader). In that case the list of characteristics is z, x, y with particular values for each; thus, five units north, 2 units west, and 7 units out would be described as 5, -2, 7, 15. These can be placed in a vector for mathematical manipulations; that vector would be [5 -2 7 15]. If the list of state characteristics is horizontal, as in [x y z], it is a *row vector*; if the list is vertical it is a *column vector*.

The notation used in quantum theory was developed by Paul Dirac and is called *Dirac Notation*, or, more casually, *bra-ket notation* (a pun on "bracket"). A row vector,

called a *bra*, is denoted $\langle x, y, z, t|$; a column vector, called a *ket*, is denoted by $|x, y, z, t\rangle$. Bras and kets are *duals* because they contain precisely the same information, the only difference being whether the information is listed horizontally or vertically. Because they are duals, the state of a quantum system can be described by either, but it is common for quantum states to be described as kets unless a mathematical operation requires the orientation to be considered. Following that convention, we will use kets to describe quantum states.

Thus, the notation below is used for bras and kets.

BRA	*KET*
$\langle x, y, z\| = [\ x\ y\ z\ ]$	$\|x, y, z\rangle = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

The dimensions of a vector are expressed as the number of rows times the number of columns: Nx1 for a column vector (ket) with N elements, and 1xN for a row vector (bra) with N columns. The bra and ket above are 1x3 and 3x1 vectors.

It is clumsy to have to write out the list of state characteristics each time the vector is referred to, so a more compact notation is used. The list of variables in the quantum state might be referred to by a symbol, often a Greek letter like ψ (spelled "psi," pronounced "sigh"). So, for example, we might state that "psi is defined as the list of quantum states "x y z ", written as the ket $|\psi\rangle = [\ x\ y\ z\ ]$. In this case we can refer to the vector as a ket $|\psi\rangle$ (a column vector) *or* as a bra $\langle\psi|$ (a row vector). Isn't that easier?

Tensor mathematics is the foundation of quantum analysis. It is a difficult subject—Einstein had trouble with it—but fortunately we won't have to immerse ourselves in it. However, two simple vector operations from tensor mathematics will be used on occasions. Suppose we have two quantum state vectors $|\psi_1\rangle = [x_1\ y_1\ z_1\ ]$ and $|\psi_2\rangle = [x_2\ y_2\ z_2\ ]$ (both expressed as kets, or column vectors, but written here as rows to conserve space). The *dot product* of a bra and a ket, also called the *inner product*, is

$< \psi_1 | \psi_2 >$ (bra times ket); it is calculated as $< \psi_1 | \psi_2 > = x_1x_2 + y_1y_2 + z_1z_2$.[1] The dot product is simply *the sum of the cross products* of the state characteristics. If the dot product is formed by a vector with itself, as in $< \psi_1|\psi_1 >$; it is the sum of squares of the state variables.

Another operation is the *tensor product*. This is the product of two kets, denoted officially as $| \psi_1 >\otimes| \psi_2 >$ but often written as $| \psi_1 >| \psi_2 >$ or as $| \psi_1\psi_2 >$. The tensor product of two vectors is another vector formed by multiplying the first state value (x_1) in the first vector by the entire second vector ψ_2, then the second state in the first vector (y_1) is multiplied by ψ_2, and so on. Thus, the tensor product of two 3x1 kets is a 3x3 matrix listing all of the possible products of the state variables.

$$| \psi_1 >\otimes|\psi_2 > = \begin{bmatrix} x_1| \psi_2> \\ y_1| \psi_2> \\ z_1| \psi_2> \end{bmatrix} = \begin{bmatrix} x_1x_2 & x_1y_2 & x_1z_2 \\ y_1y_2 & y_1z_2 & y_1x_2 \\ z_1x_2 & z_1y_2 & z_1z_2 \end{bmatrix}$$

The purpose of a tensor product is to list out al the interactions between state characteristics. It also represents the combination of two quantum states, so if $| \phi >$ is the ket for one quantum state and $| \eta >$ is the ket for another quantum state, the tensor produce $| \phi >\otimes|\eta >$ describes a combination of the states. This is called the *Product Rule for Composite States*; we will use it often.

A more detailed discussion of Linear Algebra—the mathematical foundation of Tensor Math—is in the Technical Appendix to this series.

Superpositions of Quantum States

Clearly there can be many states of a quantum system, perhaps an infinite number, each depending on particular values for the state characteristics. For example, our spatial and time measurements are all on the real line (so if t_0= 1 and t_1=2, any number between them (say, t = 1.3478254) is also a point in time. Thus, there are an infinite number of possible values on the time axis, on the x-axis, and so on. But not all

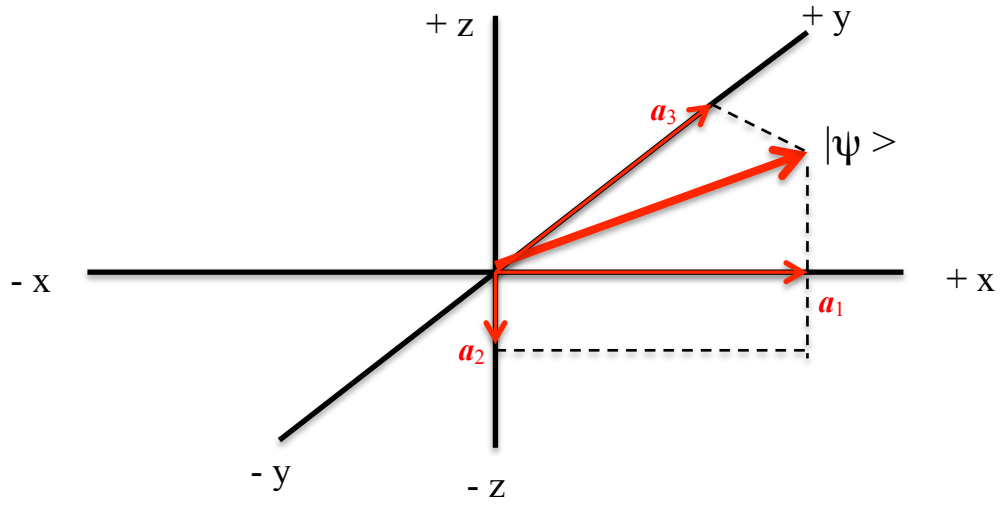
[1] In linear algebra—a branch of mathematics closely related to tensor math—the dot product is called the *inner* product.

quantum states are continuous—some are *quantized*. For example, the radius of an electron's orbit around a nucleus can occur only in discrete values, and the energy of an electron can change only in discrete amounts. And—mystery of mysteries—even time is quantized in units called *Planck time*. These units are so small that we never see them as discrete, just as we don't see a movie as a series of discrete pictures.

Suppose that we allow only 3 basis states with kets $|\psi_1>$, $|\psi_2>$, $|\psi_3>$. A superposition of those 3 states is $|\psi> = \boldsymbol{a}_1|\psi_1> + \boldsymbol{a}_2|\psi_2> + \boldsymbol{a}_3|\psi_3>$. The $\boldsymbol{a}$'s are called *amplitudes*. The basis vectors for the quantum system are the following 3x1 vectors:

$$|\psi_1> = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad |\psi_2> = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad |\psi_3> = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

These are called *unit vectors* because they have a 1 in one spot and zeros elsewhere. Unit vectors are important because they establish the axes in a vector space (in this case, a 3-dimensional vector space). Such a vector space is shown below as the rectangular axes ψ_1, ψ_2, ψ_3 of a 3-dimensional Cartesian space.



Three Dimensional Vector Space
With Quantum States and Superposition

The three vectors a_1, a_2, and a_3 show the magnitudes (amplitudes) in the z, x, and y directions. The heavier red arrow is the net vector for the particular quantum state $|\psi> = (\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3)$. Thus, the state $|\psi>$ is the 3x1 vector below:

$$|\psi> = \boldsymbol{a}_1|\psi_1> + \boldsymbol{a}_2|\psi_2> + \boldsymbol{a}_3|\psi_3> = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \end{bmatrix}$$

The values $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$ measure the *amplitudes* of the states to which they are attached. From *Born's Rule* (see Part 1) we know that the squared absolute value of an amplitude is the probability of that state occurring if the system is measured, that is $|\boldsymbol{a}_3|^2$ is the probability that if $|\psi>$ is measured, ψ_3 will happen and the other states won't happen. We know that probabilities must add to 1, so $|\boldsymbol{a}_1|^2 + |\boldsymbol{a}_2|^2 + |\boldsymbol{a}_3|^2 = 1$.

We will discuss the spin of a particle later, but spin provides an easy example. Spin can be up or down, denoted by ↑ and ↓, respectively. Suppose that spin along a particular axis is the only state characteristic. Then the state $|\psi>$ has one state characteristic with two possible values, ↑ and ↓. Suppose also that there are equal probabilities for each spin state, so $|\boldsymbol{a}_1| = |\boldsymbol{a}_2| = \sqrt{½}$ are the amplitudes and $|\boldsymbol{a}_1|^2 = |\boldsymbol{a}_2|^2 = ½$ are the probabilities. Then the superposition of spin states is $|\psi> = \sqrt{½}(|\uparrow> + |\downarrow>)$. We will often refer to a superposition as a *probability wave* because any superposition of quantum states is a superposition of the Scroedinger probability waves for those states.

The superposition of basis states is itself a quantum state and it must obey the rules of quantum mechanics. In quantum physics *all* basis states occur simultaneously while in a superposition. But when a measurement is made of the quantum state, only one basis state will occur. Repeated measurements will show that the i^{th} basis state ψ_i results the proportion $|\mathbf{a}_i|^2$ of the time.

Schroedinger's Cat provides a popular example (see Part 1): the cat has two states, |alive> and |dead>, with probability $|\boldsymbol{a}|^2$ of |alive> and probability $(1-|\boldsymbol{a}|^2)$ of |dead>. The superposition is, then, $|\psi> = \boldsymbol{a}|\text{alive}> + (1-|\mathbf{a}|^2)|\text{dead}>$, where ψ is the cat's unknown quantum state. The superposition says that while the cat's box is closed we can think of the cat as *both* alive and dead, each with its associated probability: hence

prior to a measurement of the quantum state, all possibilities coexist. When the box is opened and we measure the cat's state, the superposition disappears and only one state—"dead" or "alive"—exists; all other possible states vanish; the *probability wave* describing the cat's state "collapses" to the observed state.

The Schroedinger's Cat parable has two basic interpretations. *Philosophical Realists* (Einstein et al.) say that the cat's state was established before the box was opened—the cat was "really" either alive *or* dead—and we only see the reality when we open the box. The *Copenhagen Interpretation* (Bohr et al.) is that the cat is really *both* dead *and* alive; when we open the box we force nature to make a decision. So the measurement caused the result!

Superpositions also underlie the interpretation of the interferometer "experiments" in Part 1. These showed that when a photon can take one of two paths it takes *both* paths and creates destructive and constructive interference *unless* a measurement of the path taken is made, at which time the interference disappears and the photon behaves as if it could only have taken the detected path.

To summarize, if a number of basis states *can* occur, a superposition of those states *will* occur until a measurement is made. The superposition is obtained by multiplying each basis state vector by its amplitude, then adding the results together. The squared amplitudes are interpreted as the probability of occurrence of each basis state. Note that the amplitudes can be either a real or a complex numbers; for example, $\boldsymbol{a} = \sqrt{½}$ is a real number amplitude with probability $|\boldsymbol{a}|^2 = ½$; but $|\boldsymbol{a}| = i\sqrt{½}$ is a complex amplitude ($i = \sqrt{-1}$) though it has the same probability of ½. Thus, a complex amplitude still implies real probabilities. The use of complex numbers will be downplayed here, but what it really means is that the superposition is wavelike.

As noted above, a superposition is a Schroedinger Probability Wave (simply *probability wave*) combining the probability waves of the individual basis states. Before a measurement is made all possible states will simultaneously exist, but if a measurement is taken only one state will be seen. The probability wave is said to collapse to the single observed state.

Example: Polarized Sunglasses

A simple example of quantum superposition is the effect of polarized sunglasses on light received by the eye. Unfiltered light has rays oriented in all directions: some arrive vertically polarized (north-south), some arrive horizontally polarized (east-west), others arrive polarized NNE-SSW, and so on; all polarities arrive. However, when we see reflected light—like the sun reflecting off of water—the polarity is mostly in one direction (horizontal) because a reflective surface tends to absorb vertically polarized light. We see the abundance of horizontal polarity as glare. The role of polarized sunglasses is to reduce the glare by redistributing the light's polarity toward the vertical, so that we get both vertical and horizontal polarities.

Light received by the eye can be treated as a superposition of two basis states: vertical polarity, ket $| V >$, and horizontal polarity, ket $| H >$. The superposition is $| \phi > = \cos(\theta)| V > + \sin(\theta)| H >$ with θ as the angle of polarization from the vertical and ϕ as the angle of final polarity.

Suppose that the glare has a polarity of 75 degrees from vertical; that is, it arrives at an angle $\theta = 75°$ from the vertical (15° from the horizontal): almost but not quite horizontal. In that case, $| \psi > .26| V > + .96| H >$ is the superposition of the light polarities. The proportion of light received that is vertically polarized is $.26^2 = .068$ or 6.8 percent, and the remaining 93.2 percent of light is horizontally polarized. That hurts!

Suppose that you go to iEYE, your glasses store, to get sunglasses polarized at 40° from vertical. The mathematics tells us that now 58 percent of light is vertically polarized and 42 percent is horizontally polarized. The eye is no longer overloaded by one polarization. Isn't that much more comfortable?

System Measurement

The interferometer experiments in Part 1 revealed that the mere observation (i.e., measurement) of a quantum system *determines* the result. Note the language: measurement does not *reveal* the result, it *determines* the result. No longer are we in the classical world where an experiment is started and its results crank out independent of the observer's actions. Now the observer is part of the quantum system. To refresh our memory, we'll review some of the findings in Part 1.

First, if there is no measurement on a quantum system, all possible basis states occur simultaneously. Recall the "Two-Path Experiment." A photon can follow either the Upper Path or the Lower Path. The probability that it will take each path is ½, so if a photon stream (light beam) is emitted, half of the photons take one path and half take the other path. We don't know which path an individual photon takes, but we know that, on average, half of the photons take each path. We found in the Two-Path Experiment that a photon is both a particle *and* a wave: a particle because it causes one detector to click, a wave because it never arrives at the other detector due to destructive interference. If we don't measure which path a photon has taken, the photon is a wave taking *both* paths. That is the only way we can explain the constructive and destructive interference.

Our "Which-Path Experiment" revealed something even stranger. If we detect the path a photon is on *after* it has already started on a path—even without disturbing it in any way—it *always* behaves as a particle. It is as if the photon, knowing that it has been caught on one path, can rewrite its history and never have been on the other path. The observer is part of the quantum system, not independent of the quantum world—by observing the world we change the world!.

Our "Delayed-Choice Which-Path Experiment" revealed something stranger yet. Even after the photon has done all its work and is on its way direct to the final detectors, the observer's decision to measure the photon's path will affect the results; the results of the Which-Path Experiment occur even when detection is activated after the photon has made all of its decisions. It is as if the photon stops, goes back to the starting point, then it goes onto the path that was detected.

The moral is that *for an unmeasured system, all possible states occur simultaneously, with the probability of each state measured by (the square of) that state's amplitude. For the measured system, only the measured state exists and all other states vanish even though we "know" that they should still exist.*

Basic Rules of Quantum Systems

- A *basis state* of a quantum system describes the quantum state that exists with a specific set of values for the state characteristics; a basis state is indicated by a ket $| s_i >$ where s_i is the i^{th} basis state

- A superposition of basis states—defined as the sum of basis states, each multiplied by its amplitude—is a quantum state that acts as a wave. A superposition of n states is $| \psi > = \mathbf{a}_1 | s_1 > + \mathbf{a}_2 | s_2 > + \ldots + \mathbf{a}_n | s_n >$ where $\mathbf{a}_i$ is the amplitude of the i^{th} basis state and $| \mathbf{a}_i |^2$ is its probability.

- If no measurement of a system's quantum state is made, all basis states of the quantum system occur simultaneously.

- If a system's quantum state is measured only one quantum state exists (the one measured) and all other quantum states vanish.

- Repeated experiments on a quantum system—with measurement—will each result in a different basis state being observed. The frequency of those basis states is the probability of occurrence.

Quantum Spin

Spin Basics

Wolfgang Paulii was the first to postulate that a particle had a property he named *spin*. This property was required to explain the number of electrons allowed in each shell of an atom. Only later was spin confirmed by experiments. We all understand the concept of spin but a specific metaphor might solidify this understanding.

Our visible universe is one of three spatial dimensions: each point in space is represented by a position on the z-axis, a position on the x-axis, and a position on the y-axis. Consider a person on a geosynchronous satellite at a fixed position above New York City. He might define the z-axis as running through the Earth's North and South poles, the x-axis as also running through the earth's center but perpendicular to the z-axis in the East-West direction, and the y-axis as also running through the earth's center but in a direction toward him or a way from him—in the "In-Out" direction. That is his spatial orientation, and every spot in the universe can be plotted as a point (z, x, y) in that space.

Now suppose that our astronaut is moving away from the Earth as he drifts to the southwest in his initial spatial frame. He will see his southward motion as the planet spinning in a northerly motion along the z-axis, his westward motion as Earth spinning easterly along the x-axis, and his backward motion as Earth moving away along the y-axis. The axis of Earth's perceived spin will be determined by the astronaut's motion. For example, he might view Earth as spinning northeasterly at an angle of 30 degrees. Earth's spin is in the eye of the beholder!

Of course, Earth actually does spin, and its spin creates an electrical current that, in turn, creates a magnetic field. The electrical current arises from the outer core of molten metal flowing relative to the spinning surface. This makes Earth a magnetic dipole with a magnetic field running from its magnetic South Pole to its magnetic North Pole.

Just as the astronaut sees Earth as spinning in a northeasterly direction, a physicist sees a particle as spinning in a z, x, y coordinate system. And just as the astronaut knows that there is a magnetic field around Earth due to its spin, so a physicist knows that the particle's spin creates a magnetic field around a particle. The particle, like the Earth, is a

magnetic dipole with lines of magnetic force running between a south pole and a north pole along the spin axis.

The spin-induced magnetic properties of a particle are an important characteristic. At the level of the physicist in the lab, it allows external magnetic fields to be used to direct particles in desired directions by passing the particles through properly prepared magnets. It is also what allows particle accelerators to determine what type of particle is emitted by particle collisions. For example, an electron and a positron have opposite spins and, therefore, opposite magnetic fields and opposite polarities. If a photon decays into an electron and a positron while passing through a magnetic field, the electron veers one way and the positron veers the other way. This is how the positron—a particle predicted by Paul Dirac in the 1930s—was discovered.

Before delving into spin itself, it's worth clarifying that particles don't actually spin. In the early days of spin theory (just after discovery of a particle's magnetic field but before politicians coopted the term) it was thought that particles did spin and that this accounted for their magnetic properties. We now know that they don't spin because particles are really waves—probability waves. But their magnetic properties make it seem as if they do spin. So "spin" has been permanently attached to the list of particle characteristics.

Spin State Characteristics

Spin is the angular momentum of the particle around a specific axis. With three spatial axes, the total spin of the particle will be some combination of the particle's angular momentum around each axis. Thus, there is spin up or down along the z-axis (S_z), spin right or left along the x-axis (S_x), and spin in or out along the y-axis (S_y). Like all quantum characteristics, spin is a quantized characteristic measured in discrete units: spin can occur only in integral values of those units. The units for spin are $h/2\pi$ (called the *reduced* Planck Constant). These three spins are conjugate variables subject to the Uncertainty Principle: if one spin (say S_z) is measured precisely, the other two spins cannot be measured. Thus, discussions of spin measurements typically focus on one spin—the z-axis spin, S_z.

Spin directions can be either positive or negative, and spin can only take integral and half-integral values: possible spin numbers are 0, ±½, ±1½, ±2, ±2½, and so on. This last attribute is extremely important and we will discuss it a bit later.

Spin *around* the x-axis (z-spin, or "vertical spin") is classified as "up" or "down," spin *around* the z-axis (x-spin or "horizontal spin") is "right" or left," and spin *around* the y-axis is "in" or "out." The kets for these spin basis states are given in the table below.

Spin Basis States

Axis	Kets	Description
z	$\lvert\uparrow\rangle$ or $\lvert\downarrow\rangle$	"UP" or "DOWN"
x	$\lvert\rightarrow\rangle$ or $\lvert\leftarrow\rangle$	"RIGHT" or "LEFT"
y	$\lvert in\rangle$ or $\lvert out\rangle$	"IN" or "OUT"

Any spin basis state can be derived as a superposition of other basis state. The table below shows some basis state-superposition equivalences assuming amplitude $\sqrt{½}$, that is, probability ½.

Spin Superpositions

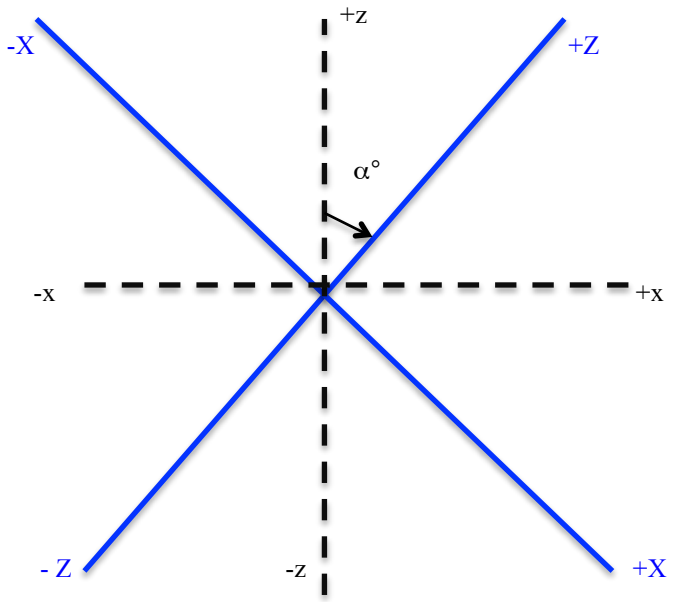
Axis	Superposition State
z	$\lvert\uparrow\rangle = \sqrt{½}(\lvert\rightarrow\rangle + \sqrt{½}\lvert\leftarrow\rangle)$ $\lvert\downarrow\rangle = \sqrt{½}(\lvert\leftarrow\rangle - \sqrt{½}\lvert\rightarrow\rangle)$
x	$\lvert\rightarrow\rangle = \sqrt{½}(\lvert\uparrow\rangle + \sqrt{½}\lvert\downarrow\rangle)$ $\lvert\leftarrow\rangle = \sqrt{½}(\lvert\downarrow\rangle - \sqrt{½}\lvert\uparrow\rangle)$
y	$\lvert in\rangle = \sqrt{½}(\lvert\uparrow\rangle + \sqrt{½}\lvert\downarrow\rangle)$ $\lvert out\rangle = \sqrt{½}(\lvert\downarrow\rangle - \sqrt{½}\lvert\uparrow\rangle)$

Note: y-axis spin is a complex number

Thus, both UP and DOWN spins are superpositions of a right spin and left spin, and RIGHT and LEFT spins are each superpositions of UP and DOWN spins. These characteristics are important in problems requiring spin mathematics, but will not detain us now.

As noted above, spin is around a directional axis that may or may not be along a single axis. For ease of exposition we focus on z-axis spin and x-axis spin, ignoring the complex-valued y-axis spin.

Consider the figure below where the x and z axes are rotated clockwise around the y-axis at an angle of $+\alpha°$ from the vertical to form a new x-axis and a new z-axis. The new z-axis represents the direction of an UP or DOWN spin, while the new x-axis represents the direction of a RIGHT or LEFT spin. There are some interesting results arising from different rotation angles.



In the next section we will discuss the spin characteristics of specific types of particles. There we will see that particles with ±½ spin are of great importance in our everyday lives: they are the building blocks of all matter. So it is worth seeing how spin ±½ particles are related to the angle of rotation, α.

Spin—like all quantum characteristics—exists in a superposition of all spin states; it is, therefore, driven by probabilities. The squared amplitude of each spin state describes the probability of that state occurring. That probability, it turns out, depends on the angle of rotation. This is shown in the table below.

Angle of Rotation and Spin ±½ Probability

α°	| spin >	P(+½)	P(–½)
000°	| ↑ >	1.00	0.00
045°	| ↗ >	0.85	0.15
090°	| → >	0.50	0.50
135°	| ↘ >	0.15	0.85
180°	| ↓ >	0.00	1.00
225°	| ↖ >	0.15	0.85
270°	| ← >	0.50	0.50
315°	| ↙ >	0.85	0.15
360°	- | ↑ >	1.00	0.00

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If there is no axis rotation (the spin is UP or DOWN along the original z-axis), the spin *must* be positive; if there is a 180° rotation (the original +z and –z axes are reversed), the spin *must* be negative. But as the rotation angle increases from 000° to 360° the probability of a positive spin decreases and the probability of a negative spin increases. And as the angle of rotation continues from 180° back up to 360° the probability of an UP spin rises. Thus, a particle with ½ spin can take on any spin direction (+ or -) as the angle of axis rotation goes from 000° to 360°. (Note, for later use, the strange minus sign before the ket at 360°.)

For everyday objects a 360° axis rotation returns the object to its original position: point a pencil straight up and rotate it clockwise 360°; it returns to its original state. This is true of some particles as well—*bosons*, the force-carrying particles. But rotation of some subatomic particles—fermions, the particles of matter—confounds our everyday understanding because *a 720° rotation is required to to return a fermion to its original spin sta*te.

A hint of this is in the table above: when a full clockwise axis rotation is completed the spin state ends up at –|↑ > (the antistate of |↑ >) rather than |↑ >: the ket for spin direction is positive until 360° is reached—one full rotation—after which the ket is preceded by a negative sign. The original positive ket is not restored until two full rotations (720**°),** when the cycle begins again. This double-rotation cycle is yet another way that quantum mechanics confuses and confounds.

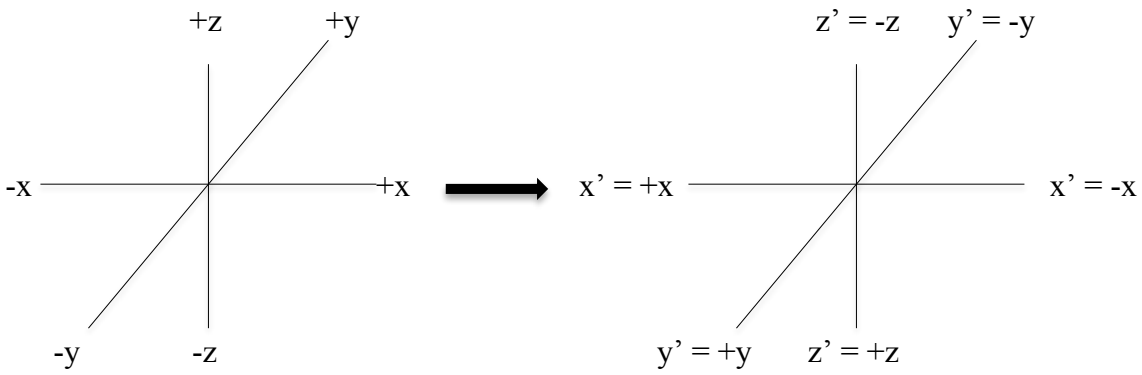
The table below shows the spin state of fermions for two full rotations (720°). All states in the second rotation are antistates of the first rotation states. Thus a 585° rotation has state – | ↙ >, not the | ↙ > associated with a 270° rotation.

Spin-½ Particle States over 720° Rotatin

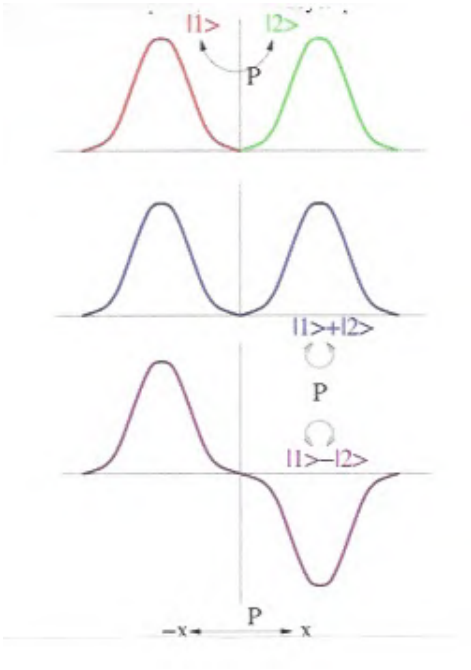
α°	|Spin >	α	|Spin
000°	| ↑ >	405°	– | ↗ >
045°	| ↗ >	450°	– | → >
090°	| → >	395°	– | ↘ >
135°	| ↘ >	540°	– | ↓ >
180°	| ↓ >	585°	– | ↙ >
225°	| ↙ >	630°	– | ← >
270°	| ← >	675°	– | ↖ >
315°	| ↖ >	720°	| ↑ >
360°	– | ↑ >		

What does a negative ket represent? Consider the superposition of states at 90° rotation and at a 450° rotation, that is | → > and –| → >; with equal probabilities, the superposition is | ψ > = √½(| → > – | → >) = 0: for a spin-½ particle a second axis rotation leads to nullification of the probability wave: the superposition has destructive interference and can not exist.

At a deeper level, the negative ket reflects a property called *spin* that affects the probability wave function when there is a "spatial inversion." A spatial inversion occurs when the spatial axes of the system (x, y, z) "flip" to a new system (x', y', z') with x'= -x, y' = -y, and z' = -z, as below



The effect of a spatial inversion is to transform a state to its antistate, thereby inverting its probability wave, as shown below. The top wave function is an original probability wave that has the same form for both positive and negative values of the distance along the x-axis. The second wave function represents a *positive parity transformation* leaving the function unchanged. A positive parity transformation is denoted by a positive (or unsigned) ket. The third wave—a *negative parity transformation*—is quite different: the +x side of the wave is inverted from its original (top) form. The shift from a negative to a positive side of an axis is a spatial inversion represented by a negative ket.



Parity Transformations

The inversion might not be along a spatial axis. In the case of axis rotations, it is in the angle of rotation. In fermions, the first 360° rotation of α leaves the wave function unchanged. But when α hits that 360^{th} degree, an inversion occurs and during the second full rotation the state flips to its antistate, turning (say) $|\nearrow>$ into $-|\nearrow>$ which is simply $|\swarrow>$.

Spin Complementarity

Part 1 reviewed the long running *Complementarity Debate* between Nils Bohr and Albert Einstein. Heisenberg's Uncertainty Principle said that when state characteristics are conjugate variables (i.e., are complementary), precision in measurement of one characteristic (momentum) implies imprecise measurement of the conjugate characteristic (velocity). Einstein rejected the concept of complementarity, believing that physics is deterministic so all variables have characteristics that can be precisely measured. Bohr won the complementarity debate and his view has become widely held.

The Uncertainty Principle applies to spin as well as other state characteristics. The more precisely spin along one axis is measured, the less precisely it can be measured along the other two axes. If spin along the z-axis, x-axis, and y-axis is denoted S_z, S_x, and S_y, then precise measurement of S_z means that S_x, and S_y can't be measured. If one spin is measured a precise answer is given, but if you then try to spin on another axis, spin on the first axis reverts to a superposition.

In the next section we address the properties of quantum particles. Of particular importance is the distinction between fermions and bosons. That distinction, we will see, turns on the different rotational spin properties of the two particles.

Spin Properties

- A particle's spin is a quantum state representing the *angular momentum* of its rotation around its spin axis.
- Spin determines the *magnetic dipole moment* of a particle and, therefore, its motion when affected by an external magnetic field.
- The spin state of spin-½ particles (*fermions*) is random. P(spin = +½) falls from 1 to 0 as the angle of rotation (α) of the z-axis from vertical increases from 0° to 180°, then it rises from 0 to 1 as α rises from 180° to 360°.
- For bosons—force-carrying particles with spin-0—a 360° rotation returns the spin quantum state to its original state, but for fermions—spin-½ matter particles two full rotations (720°) is required to return to the original state; during the second rotation the spin state is negative due to spatial inversion.
- This strange property of fermions (spatial inversion) is central to the *Pauli Exclusion Principle* outlined in the next section.

Quantum Particles: Bosons and Fermions

Macroscopic things are each unique because they are complex: no two snowflakes are identical; no person's fingerprints are identical to another's. But elementary quantum particles of a kind are all exactly alike: one electron is identical to every other electron, and so on. But though they are the same they are not *identical* particles in the sense of quantum theory.

Identical Particles

In everyday language identical particles would have the same physical properties of mass, spin, and electric charge. In this sense all electrons are "identical": they all have spin-½, mass of 0.511 MeV/c^2 (mass is in units of mega-electron-volts divided by the squared speed of light) and charge -1. But two "identical" electrons can be in different quantum states (for example, up or down spin) so in a quantum sense they are not identical.

The issue of identical particles in quantum physics is whether two "identical" particles can be swapped (or exchanged) without affecting the system's quantum state, i.e., the wave function. If they can be swapped without affecting the quantum state, they are said to have *symmetric* states; if a swap changes the quantum states, they are in *antisymmetric* states.

Suppose that there are two particles, A and B, at respective positions x_1 and x_2 on the x-axis. The electrons have states $|\psi_A\rangle$ and $|\psi_B\rangle$. Suppose also that two joint states are formed by combining those ⊗particles: $|\psi_A\rangle\otimes|\psi_B\rangle$ is the state when A is at x_1 and B is at x_2, and $|\psi_B\rangle\otimes|\psi_A\rangle$ is the state when A is at x_2 and B is at x_1; their positions are reversed. These are called *product states* because a combination of two quantum states is the tensor product of the states.

Are these states identical? As noted above, identical particles have the property that if they are swapped with each other, the quantum state is unchanged, meaning that the two-particle system's wave function is unaffected: *two particles are identical if* $|\psi_B\rangle\otimes|\psi_A\rangle = |\psi_A\rangle\otimes|\psi_B\rangle$; superposition $|\psi\rangle = \sqrt{½}(|\psi_A\rangle\otimes|\psi_B\rangle + |\psi_B\rangle\otimes|\psi_A\rangle)$ shows that when the particles are identical $|\psi\rangle = 2\sqrt{½}(|\psi_A\rangle\otimes\psi_B\rangle) = 2\sqrt{½}|\psi_B\rangle\otimes\psi_A\rangle$:

the wave functions are identical.[2] Thus, both product states are the same if the two particles are interchanged with no change in the probability wave function.

Now let's look at the second possibility—the two product states are not the same, that is, $| \psi_A >\otimes|\psi_B > \neq | \psi_B >\otimes|\psi_A >$. The states are antisymmetric so $|\psi_A >\otimes|\psi_B > = - | \psi_B >|\psi_A >$; now $| \psi > = 1/\sqrt{2}(| \psi_A\psi_B > - |\psi_A\psi_B >) = 0$. The effect of the antisymmetry is to invert the wave function and create destructive interference. The destructive interference means that the two particles can not share the same quantum state. Another way of describing antisymmetry is that the particles avoid each other so that they don't come together and get cancelled out.

The Spin Statistics Theorem

Why does this matter? The reason is that it distinguishes two fundamental types of matter: fermions and bosons. We saw earlier that fermions have half-integer spins, i.e., spin of ½, 1½ , 2½, etc.; *all known fermions are spin-½ particles*. Bosons have integer spins, i.e., 0, 1, 2, etc.; *all known bosons have spin-1*.[3] But now we have another difference: bosons are identical particles and, as such, they can share the same quantum state and can be packed closely together; bosons are "gregarious." Fermions, on the other hand, can never share the same quantum state and, as such, they avoid each other; they are "antisocial." Fermions of the same type (say, electrons) develop this avoidance mechanism by having the same electric charge: all electrons have a charge of -1 so electrons repel each other; all protons have a charge of +1 and protons repel each other.

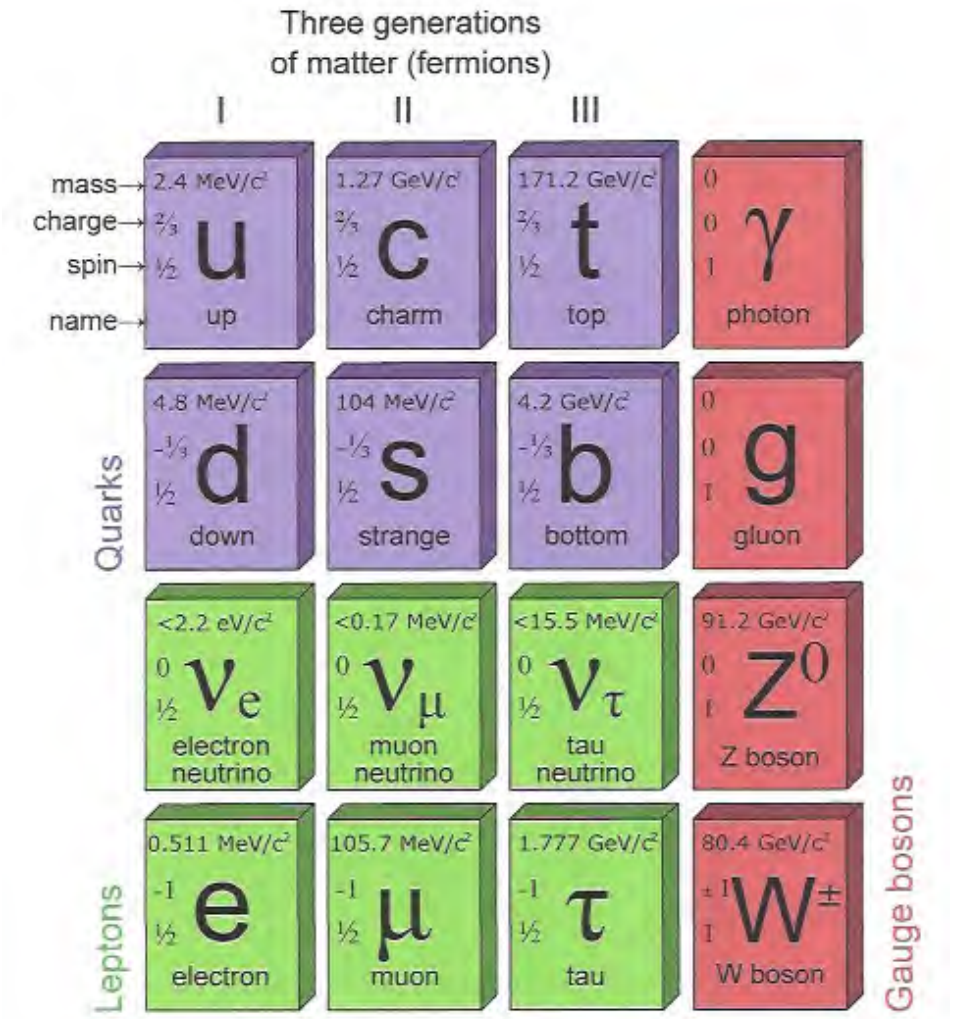
This is summarized in the s*pin statistics theorem:* collections of like-particles in symmetric states (spin-0 or spin-1 bosons) leave the system's probability wave function unchanged, while particles in antisymmetric states (spin-½ fermions) invert the wave function. This is a fundamental distinction in the Standard Model of Elementary Particles.

The Standard Model of Elementary Particles

The table below shows the sixteen elementary particles in the Standard Model. A seventeenth elementary particle might now be added: the Higgs boson—long predicted

[2] Multiplication by a constant like √2 makes no difference to the wave function.
[3] The recently discovered Higgs boson has spin-0.

but only very recently discovered—plays an essential role in determining a particle's mass and, therefore, gravity. Each particle has a triplet of characteristics—mass, spin, and charge—that make that particle unique (but not necessarily *identical*.)



The Standard Model

The properties of the elementary particles of matter that make up you, me, trees, dogs, and stone, were studied by Enrico Fermi and are called *fermions*. All fermions are spin-½ particles, are antisymmetric and, therefore, are antisocial. There are twelve fermions: six quarks that make up the proton and neutrons in the atomic nucleus, and six leptons that define all other fermions.

Quarks are the lighter (less massive) fermions; the electron, the muon, and the tau are the heavier fermions. A proton is made of two up quarks and one down quark (each of a different color) giving the proton a +1 charge. A neutron consists of two down quarks and one up quark (again, each of a different color) with a zero charge. Electrons are very stable with an extremely long half-life, but muons and taus are very unstable, with an almost instantaneous half-life. For each of these heavier leptons there is a neutrino form with small mass and zero charge. Neutrinos interact with other particles so rarely as to never be seen in the act.

Bosons are *force-carrying particles*. The *photon* carries the *electromagnetic force* that makes up electromagnetic radiation ranging from very long wavelength radio waves, through visible wavelengths of light, on up to extremely short wavelength ultraviolet. radiation. The *gluon* carries the *strong force* that binds the proton to the neutron in an atom's nucleus. Both the photon and the gluon are massless particles with spin-1 and zero charge; like all massless particles they zip around at the speed of light.

The W and Z bosons carry the *electroweak force* that plays a major role in particle decay. They have mass and are both spin-1. The Z-boson has zero charge but the W-boson has charge of ±1 (allowing it to be its own antiparticle).

The Higgs boson (not on the list) carries the gravitational force and gives mass to all fermions and the W and Z bosons. It is very massive (on the order of 125 GeV/c^2) with zero charge and zero spin. Its existence is still tentative though in July, 2012 physicists using the Large Hadron Collide reported evidence of their existence.

Bosons have a number of important applications. They are the foundation of lasers because they generate coherent light beams of photons, each with the same quantum state. Because each photon's wave has the same frequency, the color of a laser light beam is pure, and the laser beam is very precise.

Bosons are also the foundation of *superconductivity*, the property of creating an electric charge with no resistance. At very low temperatures some metals, like rubidium, conduct electricity with no resistance and, therefore, no loss in energy. *Superfluidity* is another application: at extremely low temperatures Helium II is exhibits no viscosity and flows without resistance.

The Pauli Exclusion Principle

Wolfgang Pauli was the first to note that if an interchange of two "identical" particles changes the sign of the quantum states: these particles can not have the same quantum state. The consequent antisocial nature of fermions is called the *Pauli Exclusion Principle*. Matter occupies space because of the Pauli's Exclusion Principle: particles of matter cannot occupy the same place so they must spread out. The inability of fermions to occupy the same place is why we don't go through the floor when we stand or walk, why we can't put our fist through a brick wall, and so on.

The Pauli Exclusion Principle is also the foundation of chemistry. It explains the *Shell Model of the Atom* in which electrons are arranged in shells (orbits) corresponding to their energy levels. The inner shell can have two electrons, the second shell can have eight electrons, the third shell can have eighteen electrons, and so on. When the outer shell is filled, the atom is very stable, refusing to lose or gain electrons because of the high energy required to create a new outer shell.

The *Periodic Table* arises from the shell model and, therefore, from the Pauli Exclusion Principle. Each atomic number shows the number of protons in the nucleus, each with charge +1(because the atom is neutral in its normal state, this is also equal to the number of electrons, each with charge -1). Thus, atoms have no charge unless they gain or lose electrons because of an external energy kick. Stable atoms—atoms with filled outer shells, like metals and inert gases—don't chemically interact because chemical interaction requires incomplete outer shells so that an exchange of electrons can occur.

The Atom

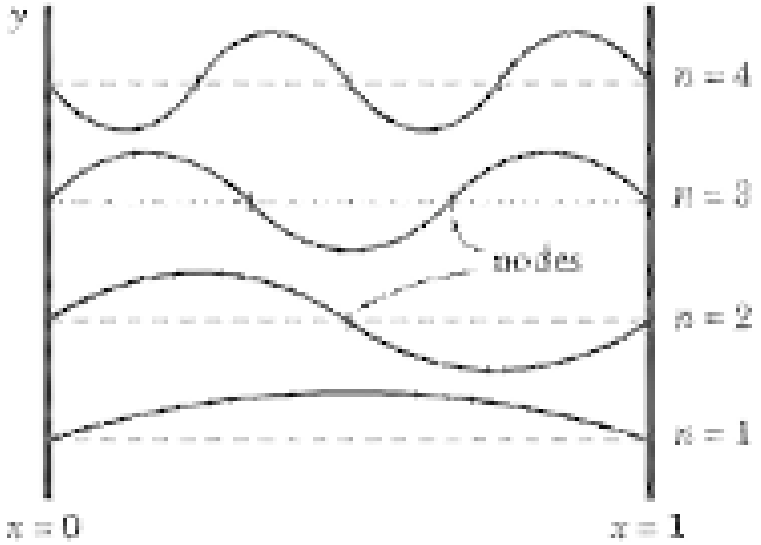
The Shell Model of the Atom

In this section we will draw out some details of the modern model of the atom. In Part 1 we saw that Ernest Rutherford's "planetary model" of the atom was rescued by Niels Bohr's insight that electrons orbit the nucleus at specific "quantized" energy levels: the greater the radius of the orbit, the higher is the electron's energy level, and an electron doesn't change to a higher or lower orbit unless there is a discrete change in its energy. Bohr's model of the atom implied a minimum energy level, preventing an electron from losing all of its energy and spiraling into the nucleus, thus destroying the atom.

Bohr's model defined the energy levels associated with each orbit. The *quantum of energy* is $E = h/\lambda$, where λ is the wavelength of the electron's probability wave and h is Planck's Constant. Bohr also found that the probability wave of an orbiting electron must be a standing wave and it must conform to energy levels associated with vibrational lengths equal to one-half λ, one wavelength, 1½ wavelengths, two wavelengths, and so on.

A standing wave is a wave that is not seen to travel in any direction except vertically. At any location it simply moves up and down as it passes through a cycle. A standing wave arises when a traveling wave is reflected backwards onto the initial wave, as in a violin string (with vibrations reflected from the endpoints tied to the violin) or a water wave that hits a seawall and reflects in the opposite direction.

Several standing waves are shown below. The bottom wave (n = 1) is for the lowest energy level (E_1), occupying the lowest orbit; the next highest energy level (E_2) is at the second orbit, and the third energy level (E_3) is at the third energy level. Because the energy quantum is $E = h/\lambda$, the associated energy levels are $E_1 = 2(h/\lambda)$, $E_2 = 3(h/\lambda)$, and $E_3 = 4(h/\lambda)$. The orbital "ring" is defined by the number (n_1 for energy level E_1, n_2 for energy level E_2, and so on). The number n is called the *primary quantum number* because it defines the quanta of energy associated with the orbital ring. We will see that there are additional energy-related quantum numbers.



Standing Wave Patterns

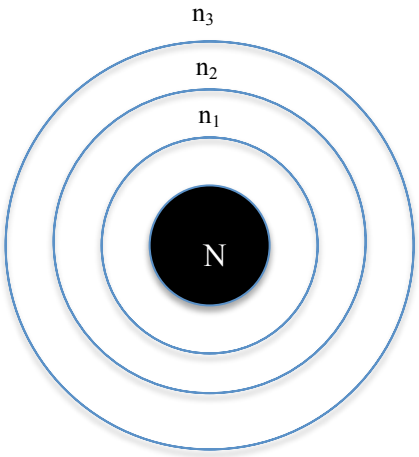
The figure above shows the standing wave patterns for electrons in the first four orbits around the nucleus. The rapidly vibrating electrons in the higher orbits have higher energy levels, but the differences in energy between adjacent orbits are in discrete quanta. An electron can jump to a higher orbit (and a more rapidly vibrating standing probability wave) if it is given a kick from an external energy source; it can fall to a lower orbit (and a lower vibration frequency) if it loses a quantum of energy to an external source.

Each of the standing waves is the result of a direct wave and a reflected wave that, when combined, move vertically (along the y-axis) but not horizontally (along the x-axis). The lowest wave (quantum number n = 1, energy E_1) is for the lowest orbit and lowest energy level: it has a length equal to one-half the direct wave's length. This standing wave is called the *fundamental wave*. The other standing waves are harmonics: the first harmonic (n = 2) has 1 full wavelength; the second harmonic (n = 3) has 1½ wavelengths; the third harmonic (n = 4) has two full wavelengths, and so on.

Note that each of the waves has stationary points on the x-axis that the wave passes through at every point in its vertical cycle; these are the wave *nodes*: the n = 1 wave has two nodes, the n = 2 wave has three nodes and so on. The stationary positions on the x-axis where a wave level is zero are called *antinodes*.

Bohr's model is the figure shown below, in which each electron orbits around the nucleus (N) in a separate orbital ring (n_1, n_2, n_3,…) chasing the electrons ahead of it on

the same path. It has the form of Rutherford's atom, with the addition of an energy quantum that keeps the orbits apart at specific distances.



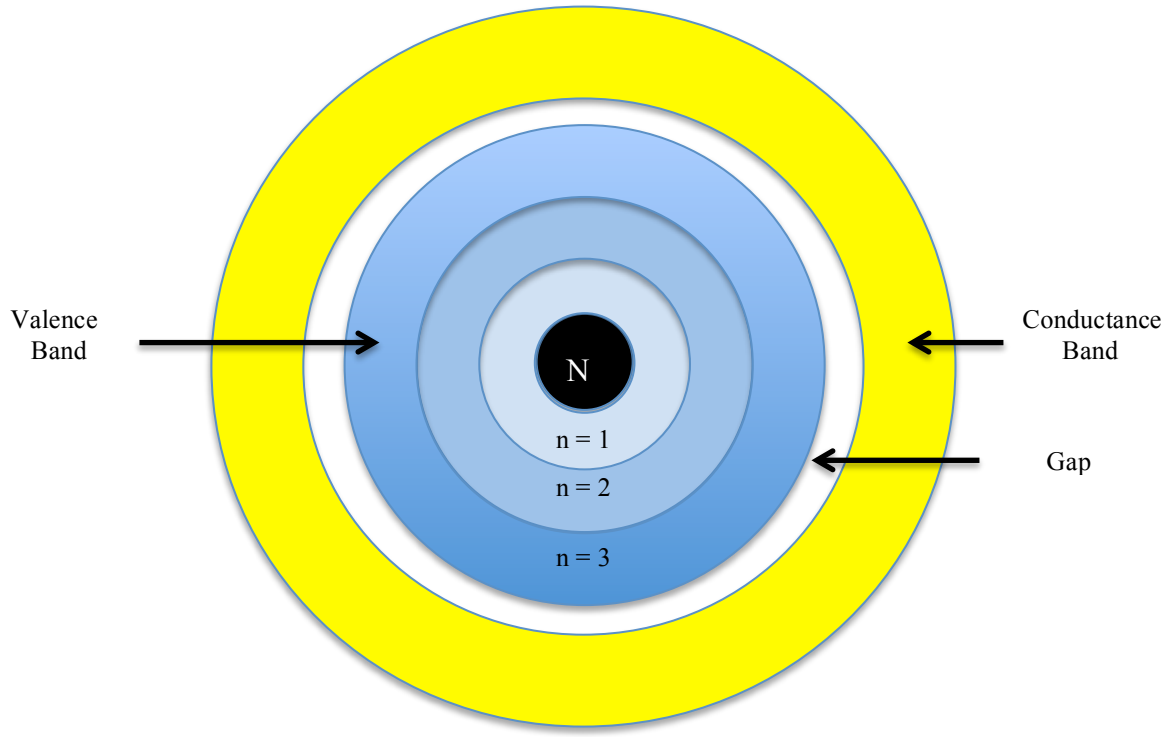
Bohr's Atomic Model

Our focus has been on a single electron. Now we look at the structure of electrons in an atom. There are three important lessons. First, Bohr's model of a hairline path for each orbit is incorrect; instead, an orbit is best described as an *energy shell*, with the electrons in each shell having very similar, but not exactly the same, energy levels. Second, there are a maximum number of electrons allowed in each shell. Third, the conduction of electricity is intimately connected to the electrons occupying the outer rings.

As to the first point, the modern view of the electron is shown below. Each energy shell contains several electrons, each with a slightly different energy level and, therefore, a slightly different standing wave. Three energy bands (shells) are shown: n=1 has the lowest energy level, n=3 has the highest. There can be higher energy shells: the highest known shell is n = 6, where some electrons for element 112 (copenricium) reside.

The highest occupied energy band (n = 3 in the diagram) is called the valence band. It is the primary source of electrons in the conduction of electricity between atoms. Outside the valence band is the conductance band. This band is typically empty until an energy boost kicks an electron out of the valence band. Electrons in the conductance band are "free electrons" that can be shared with other atoms.

There can be a gap between the valence band and the conduction band, as shown in the figure below. No electrons are allowed in the gap, so movement of an electron from the valence band to the conduction band requires an energy boost large enough to bridge that gap. The width of that gap is an important characteristic in the conductivity of electricity between atoms.



The Shell Model

The higher shells are occupied by electrons with higher energy levels. But the increment in energy level required to create a higher shell declines as the shell number increases. This is because the atom is electrically neutral with the positive charge of the protons balanced by the negative charge of the electrons. The attraction between the nucleus and a proton is exactly offset by the rotational energy of the electron, just as the attraction between the earth and moon is offset by the rotational energy of the moon. But the attraction between the protons and electrons decreases with the shell number because the electrons in outer shells are farther from the protons. Thus, it requires a larger energy

boost to move an electron from n = 1 to n = 2 that the boost required to move from n = 2 to n = 3.

The number of electrons in a shell is determined by the values of four quantum numbers *n, l, m, and s*. Quantum number *n* is called the *primary quantum number*: it defines the energy band of an electron. Quantum number *l*—the *azimuthal quantum number*—is a subband within the energy band is defined by the angular momentum of the electron and describes the orbital shape taken by the electron. For energy band *n* there can be as many as *n* of these subbands; thus the n = 3 band can have as many as three subbands. Quantum number *m* is the *magnetic quantum number*. It plays a role in the interaction between the electron and an external magnetic field. Finally, quantum number *s* is the electron's spin, with two possible values: up and down.

Pauli's Exclusion Principle, introduced above, says that no two electrons in an atom can have exactly the same quantum states, so at least one of the four quantum numbers must be different if more than one electron is to be allowed. This means that each shell can have no more than two electrons with the same energy-related quantum numbers *n, l, and m;* those two electrons must have opposite spins (*s*). As a result, in any filled shell half of the electrons are spin + ½ and the other half are spin -1/2.

Electron Configuration[4]

Shell (n)	Maximum Electrons Allowed
1	2
2	8
3	18
4	32
5	50
6	72
7	98
8	128

[4] This table shows the maximum allowed electrons. Many elements have inner shells with less than the maximum allowed. For example, copernicium, with 112 electrons, has eight shells with 2, 8, 18, 32, 32, 18, 8, and 2 electrons rather than the 2, 8, 18, 32, 50, 2 configuration associated with maximally-filled shells.

There is a simple algorithm for determining the maximum number of electrons in a shell: *the n^{th} shell can hold up to $2n^2$ electrons.* Of the $2n^2$ electrons in the n-shell, n^2 are due to the allowed energy levels *n, l, and m*; the addition of spin doubles the number of electrons allowed by the PEP. The table below shows the number of electrons allowed in each shell. Note that the number of electrons in a filled shell is always an even number. Any shell with an odd number of electrons is, by definition, unfilled.

Electron Configuration[5]

Shell (n)	Maximum Electrons Allowed
1	2
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8	128

The table above shows the number of electrons allowed in each shell. Note that the number of electrons in a filled shell is always an even number. Any shell with an odd number of electrons is, by definition, unfilled.

Electrical Conductance

The *atomic number of an element* is the number of electrons in its atom, always equal to the number of protons. Hydrogen, with atomic number 1, has one proton and one electron; that electron is in the n = 1 shell that could contain two electrons; addition of that second electron creates a helium atom. Copernicium, discovered in 1996 and only recently (2010) added to the periodic table, has 112 protons and 112 electrons with 8

[5] This table shows the maximum allowed electrons. Many elements have inner shells with less than the maximum allowed. For example, copernicium, with 112 electrons, has eight shells with 2, 8, 18, 32, 32, 18, 8, and 2 electrons rather than the 2, 8, 18, 32, 50, 2 configuration associated with maximally-filled shells.

electrons in its outer (valence) shell. Copper—an excellent conductor—has 29 protons, 35 neutrons, and 29 electrons. This means that the n = 1 band is filled with two electrons, the n = 2 band is filled with eight electrons, the n = 3 band is filled with eighteen electrons, and the n = 4 has only one electron. The n = 3 band—the outermost completely filled shell—is the valence band; it can not take on any additional electrons and it does not readily give up electrons.

The relationship between the valence and conduction bands determines whether a material is an *insulator*, a *semiconductor*, or a *conductor*. An insulator has a relatively wide gap between the two bands, requiring an extra large energy kick for a valence band electron to jump to the conductance band and initiate an electron flow. The gap between the valence and conductance bands for a semiconductor is smaller, making it easier for a valence electron to get an upward jump. For conductors there is no gap—the valence and conduction bands partially overlap, and the greater the overlap the better the conductivity; in the extreme, with complete overlap, the conductance band *is* the valence band so all electrons in the two bands can be easily conducted to other atoms.

Electrical conductance is the transfer of electrons between atoms. When an electron leaves one atom to join another, the electrical charge of both atoms changes. The "giving" atom becomes more positively charged because it has now has fewer electrons than protons, the receiving atom becomes more negatively charged because it has more electrons than protons. This imbalance means that the electron that moves is a hot potato—the receiving atom gets rid of it by passing it to another atom, and so on. At the same time, the giving atom, now positively charged, needs another electron and draws it from another atom. In this way, electrons—and their negative charges, flow throughout the conducting material.

Electrical conductivity changes the atoms in a conducting material and, therefore, changes the materials through which the electrons are conducted. Perhaps the most obvious every-day example is *corrosion*. When two dissimilar materials are joined, the transfer of electrons between the metals creates a juncture which is a third element that not only looks different but also can be a weak joint between two otherwise strong materials.

The shell model incorporates all of the quantum ferment with which we are familiar. For example, it requires "action at a distance" because each electron must find somehow its own unique quantum numbers. This means that electrons must "know" the quantum states of all other electrons in the atom. If they didn't, they would bump into each other like cars on the Los Angeles freeways, each attempting to take the exact quantum state of other electrons. Why doesn't this happen? The *prima facie* answer is that Pauli's Exclusion Principle doesn't allow two fermions to share the same quantum state. But this begs the question, "how does the PEP prevent this?" The answer is that somehow each electron "knows" the quantum states of the other electrons: without this information, it can not take on a quantum state different from all the other electrons.

Electrical conductance also is subject to quantum mechanics. We might imagine that the motion of electrons from one atom to another is that a giving atom releases an electron from its conductance band to the conductance band of the receiving atom. The probability that this will be the transmission route can be calculated and this is the most likely transition. But the route might be from the valence band or even lower to any band of the receiving atom; the deeper the giving band and the deeper the receiving band, the lower the probability of that transition route. In the case of "deep shell" transmission the electron's movement causes a series of additional movements within the two atoms that restores them to their original electron configuration until a stable result is achieved with the giving atom having one less conductance band electron and the receiving atom having one more conductance band electron.

Developments in Quantum Theory

Several developments in quantum theory and its applications are worth highlighting. The first is *quantum entanglement* in which composite particle systems (systems with two or more particles) exhibit correlations between their quantum states. The second is the relatively new field of *quantum electrodynamics* and its prediction of *virtual particles.* Finally, we discuss a connection between quantum theory and cosmology: *the role of virtual particles in the formation of dark energy, and in the inflationary view of the universe's creation and expansion.*

Quantum Entanglement

In Part 1we reviewed the Einstein-Podolsky-Rosen (EPR) criticism that quantum theory is incomplete. In that thought experiment, an electron and positron are created from a photon's decay. The two particles are then sent in opposite directions to observer A and observer B, who are one light-second (300,000 km) apart. It is known that at their creation the two particles have opposite spins, but the spin of each is not known. Quantum theory says that the spin states are probabilistic so that if, say, the z-spin of particle A is measured it might be $|\downarrow>$ in one experiment and, say, $|\rightarrow>$ in the next experiment. You simply can't know which spin state that will come up.

Suppose Observer A measures the z-axis spin of his particle and finds it is $|\uparrow>$. Quantum theory says that instantaneously particle B will take on spin state $|\downarrow>$. EPR argued that it was impossible for the information about A's measured spin state to be instantaneously transmitted to particle B; that would violate special relativity, which argues that information can never travel at a speed faster than light.

The EPR debate raised the question of entanglement. Can two particles with a known relationship between their quantum states be separated by a great distance and still maintain that relationship? If so, the correlation between their quantum states is preserved and the particles are said to be *entangled*.

EPR believed that physics obeyed two crucial laws: *locality*, meaning that interactions between particles at a distance could not occur faster than light speed, and *realism*, meaning that a particle's behavior can be predicted without disturbing it; in that

case the particle is deterministic and the probability rules of quantum mechanics don't apply. In short, the particle conforms to a "common sense" interpretation of determinism without interference.

Locality rules out "spooky action at a distance," and realism rules out the probabilistic foundation of quantum mechanics. So, EPR argue, there is something missing in quantum theory. There must be "hidden variables" affecting quantum states and if we just knew what those missing variables were, the probabilistic aspects of quantum theory would disappear; In such a case, A's particle could *only* have z-spin state $|\uparrow>$ and B's particle could *only* have z-spin state $|\downarrow>$. The EPR view is called *local realism hidden variables theory*.

Nils Bohr responded with the concept of quantum entanglement, in which two particles are formed in proximity with shared state characteristics, then separate to, perhaps, great distances. In this case, there is a probability wave for the composite particle; that joint wave describes their joint states no matter how far apart are the particles. If A measures his particle as having state $|\uparrow>$, the probability wave collapses to that state for particle A and it also collapses to $|\downarrow>$ for particle B. Instantaneous communication is not necessary with a joint probability wave.

The concept of entangled particles was novel and controversial, but it has become a tenet of quantum theory. Suppose there are two *un*entangled particles: each particle has equally probable basis states $|\uparrow>$ or $|\downarrow>$, so $|\psi>|\otimes\psi>$ is quantum state of the composite particle. Expanding this gives $|\psi>|\otimes\psi> = 1/4(|\uparrow> + |\downarrow>)(|\uparrow> + |\downarrow>)$ so there are four possible states for the two particles: $|\uparrow\uparrow>$, $|\uparrow\downarrow>$, $|\downarrow\uparrow>$, and $|\downarrow\downarrow>$, and the *Composition Rule* applies: [6] The four possible states each have probability ¼.

The Composition Rule applies when the two particles are *not* entangled. When they are entangled, as in an electron-positron pair, two of the four basis states are excluded; these are $|\uparrow\uparrow>$ and $|\downarrow\downarrow>$.[7]

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[6] The Composition Rule says that the joint state of two or more particles is either a simple product state or a superposition of simple product states. It applies only when the particles are not entangled because both particles have their own quantum state.

[7] The example given has states entangled in opposing spins. An equally valid entanglement would have the spins positively correlated—both up or both down.

A system of entangled particles, like our positron and electron, has a total spin state denoted by the superposition $|\psi> = \sqrt{1/2}(|\uparrow\downarrow> + |\downarrow\uparrow>)$. Because $|\downarrow\uparrow>$ is the opposite of $|\uparrow\downarrow>$ we can write $|\psi> = |\downarrow\uparrow> - |\uparrow\downarrow>$, so $|\psi> = 0$—the total spin of the two particles is zero. Even though we know that there is zero total spin along the z-axis, we do not know which spin state will occur until a measurement is taken. If, at measurement, one spin state appears for the first particle we know that the opposite spin must occur for the second particle.

In 1965 John Bell reported an amazing result that categorically refuted the EPR "paradox" by showing that its "common sense" view is inconsistent with the basic laws of quantum mechanics: local realism did not just require that quantum theory be incomplete, it required that it be wrong!

To demonstrate this, Bell reduced the EPR "local realistic hidden variable theory" to three main assumptions: (1) multiple particles can be quantum entangled; (2) all action is local—one particle's influence on another can not be transmitted faster than light speed; and (3) the laws of physics must be "realistic," by which they meant that the system is deterministic, not probabilistic. From these three assumptions, Bell argued, EPR conclude that there are hidden variables and that the probabilistic nature of quantum theory arises from our failure to understand the effects of those variables—determinism would be restored if we understood those effects. In short, *entanglement + locality + realism = hidden variables.*

Bell's Theorem showed that there were specific quantitative conditions for validity of this chain of reasoning. Bell's method was to assume that the EPR world is true and to calculate the relationship between spins for two particles having a zero total spin state ($|\psi> = 0$) using the EPR local realistic hidden variables theory. He derived *Bell's inequalities*, showing that the EPR world implied that the probability of agreement of spin in the two particles was bounded from below: if EPR was correct, the probability of spin agreement could not exceed a specific value.

Then he showed that if the same zero total spin system is analyzed using quantum mechanics, Bell's inequalities would be violated. So with entangled variables *either* locality is false and there is "action at a distance" *or* there are no hidden variables and

determinism is false (*or both*). Thus EPR can not be correct: *There can be no local realistic hidden variables theory.*

Bell's theorem has been modified over time but still has its original content. It has been reduced from a "slam dunk" deductive proof that EPR is false to an experimental test that it is false. All experiments thus far have found EPR false, so one of quantum theory's key characteristics—*non-locality* (instantaneous action at a distance) and *uncertainty*—have been supported.

Quantum Electrodynamics

Richard Feynman ("FineMan") was perhaps the most brilliant, quirky, and fun-loving physicist of the last half of the 20th century. On his way to the 1965 Nobel Prize he played drums for the samba in Rio's Carnival, solved the Mayan hieroglyphic code on his own, and engaged in riotous activities summarized in his popular book "Surely you're joking, Mr. Feynman!"

He also was a specialist in explaining physics is everyday language. As a member of the commission that investigated the explosion of the space shuttle Challenger in 1986. He concluded that the problem had been that cold weather caused rigidity in an O-ring designed to seal joints in the fuel system. As a result, fuel flowed past the seal and ignited from the engine's heat, starting the explosion. He demonstrated this on television by dropping an identical O-ring in ice water and showing how it remained rigid until warmed to room temperature.

Feynman's most important contribution was the field of *Quantum Electrodynamics (QED)*, for which he shared the Nobel Prize. QED addresses the interactions between quantum particles, both fermions (electrons, protons, positrons, etc.) and bosons (photons, gluons, etc.). QED is also called *relativistic quantum theory* because it marries two fields long thought distinct: special relativity and quantum theory. QED is a remarkably accurate theory of subatomic particle interactions, and as strange as the theory seems, it has never been refuted in the laboratory or in its theory.

As part of his development of QED, Feynman developed a visual approach to investigating interactions between electromagnetic particles. This contribution is called the *Sum Over Histories Approach* (SOH) to particle motion. The underlying mathematics

is called *Path Integral Analysis*. The SOH approach to quantum mechanics is a simpler and less mathematical method of assessing the behavior of subatomic particles, though it faithfully replicates the complex mathematics of path integral analysis.

According to quantum theory, a particle going from source A to target B doesn't travel in simple ways (straight lines, arcs, and so on). Instead, it takes *every possible route*, each with its own amplitude. The analysis of the paths is a superposition of all possible paths, each weighted by its amplitude, and the square of a path's amplitude is the probability attached to that path. If a_i is the amplitude of the i^{th} possible path and there are **n** possible paths (each labeled X_i, i = 1, 2, …, n), then SOH says that the path from A to B is

$$(A \Rightarrow B) = a_1X_1 + a_2X_2 + a_3X_3 + \ldots + a_nX_n$$
$$\text{with } |a_1|^2 + |a_2|^2 + |a_3|^2 + \ldots + |a_n|^2 = 1$$

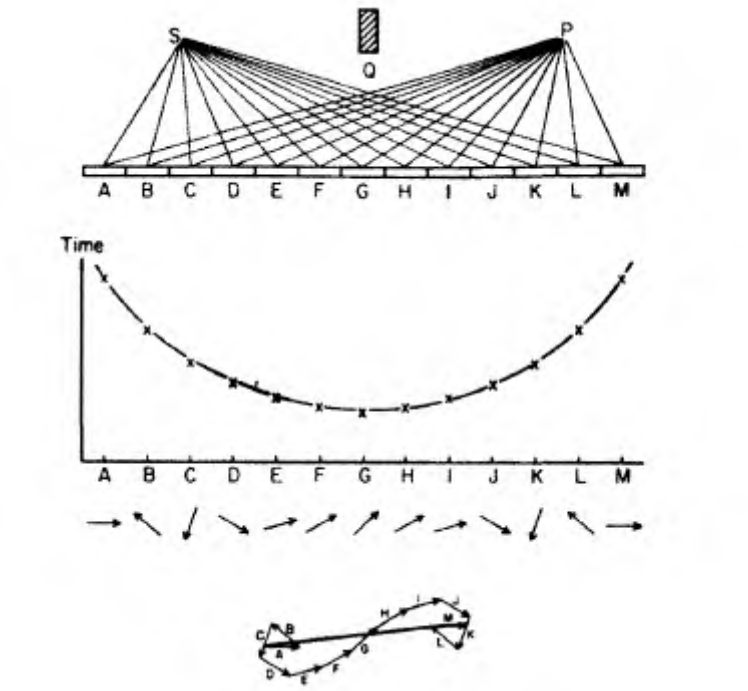
This looks very like the expected path in classical statistics, but one crucial difference is hidden in the interpretation of the amplitudes: in classical statistics the amplitudes are all positive real numbers and they *are* the probabilities; in quantum statistics the amplitudes can be negative and can be complex numbers, and the *squares* of amplitudes are the probabilities It is the complex numbers that give rise to wave-like behavior and to interference patterns.

The SOH method is a straightforward of replicating the above equation: identify all the possible paths that a particle can take to go from A to B (i.e., X_1, X_2, …, X_n), calculate the associated amplitudes (i.e., a_1, a_2, …, a_n), then draw some squiggly lines that add up to the amplitude for a particle going from A to B.

Feynman's classic example using reflection of light from a mirror is given below. This example addresses three common intuitions about light: (1) it travels in straight lines like a choo-choo train of photons, each taking the same path; (2) when it reflects, the angle of incidence is equal to the angle of reflection; (3) light takes the shortest possible path.

In the Feynman example, a photon is emitted at S, reflects from the surface of the mirror, and is detected at P. Not all photons will be detected—many will reflect in ways

that pass by the detector. The questions are "What is the probability that a photon emitted by S will be detected by P?" and "Which paths are most likely?" Feynman's method of answering this has three steps. First, identify each of the paths a photon can take from S to P. These are (by assumption) the paths labeled A through M.[8] Notice that some of the extreme paths (A, B, L, and M) are strange—those paths reflect backward at the mirror! The lesson is that light takes every possible path; those backward-bending paths are unlikely, but some few photons will tale them.



SOH Analysis of Light Reflecting From A Mirror

The second step is to start a clock running at the instant that a photon leaves S, and stop it the instant the photon is detected at P. This is a special clock, a "phase clock," also called a "Feynman clock" after its creator. In the Technical Appendix we see that it is really just a visual image of a complex number. The clock has one hand that measures the time taken on that path, measured in wavelengths of the light; that hand moves counterclockwise starting from the "15 second" position, rotating 360° (2π radians) each time the photon travels one wavelength. For example, suppose the photon is red light

[8] Not all photons will be detected—some will take paths that miss the detector. The example addresses questions about the photons that are detected.

with 450 Terahertz frequency (450 trillion cycles per second) and a wavelength of 650 nanometers.[9] The clock will do a full rotation once every 450 trillionths of a second, i.e., every time the photon travels at light speed for 650 nanometers. It is a very fast little bugger.

The position of the clock's hand at the instant it stops at the detector shows the direction, or phase, of the photon's wave at that instant. For example, on paths A and M the hand is pointing to its "15 second" starting point, indicating that the photon had just completed a full wavelength at the instant it was detected. On path G the hand is at 45°, indicating that it has completed 12.5 percent (= 45°/360°) of a cycle at the instant it is detected.

The U-shaped diagram traces out the real time taken for each path. Paths A and M take the most time because they are the longest paths. Paths G-H take the least time because they are the shortest paths. The message here is that some paths taken are shorter than others: not all paths are straight lines.

Finally, vector additions are performed on the clock hands to determine the probability that a photon will be detected and the most probability attached to each path. The result is the worm-like shape at the bottom. It is constructed as follows: copy the clock reading for path A, then at the end of that arrow add the clock reading for path B (foot of the B reading tadded to the arrow end of the A reading), then add path C's arrow to the end of path B's arrow, and so on; always maintain the directions of the vectors when you add them. Note that paths A-D and J-M both circle around with no particular direction. This means that they add little to the total probability that a photon goes from S to P. But look at paths E-I. Those head in pretty much the same northeasterly direction, adding together to make a long line (high amplitude). Thus indicating a high probability that a detected proton takes one of those paths.

The probability of a photon being detected at P is represented by the heavy line connecting the start of path A to the end of path M. Of course, you would need the underlying mathematics to put a number on that probability, but the fact that almost all of

[9] The strange starting point and the counterclockwise rotation is because the clock is showing the angle of the hand relative to the horizontal, just as if a trigonometrician were measuring the angle of a line from the center to the edge of a circle. After all, the clock is just a mechanism for visualizing complex underlying mathematics.

the length of that final arrow is due to E-I shows that most detected photons took one of those paths.

So common intuitions are *not* supported by the way light behaves: (1) light does not travel in a straight line, though that is the most probable path; (2) on every path except G the angle of reflection is *not* equal to the angle of incidence; (3) photons tend to take the shortest path, but they will travel all possible paths, some of them very strange.

This is a very creative way to tell stories without going through the very difficult mathematics. It has been shown that path integral analysis (Feynman's SOH) is fully consistent with the mathematics of quantum theory, so either method can be used; some problems are more tractable with SOH, others with standard quantum mathematics.

Virtual Particles

Einstein showed that there are three ways a photon and an electron can interact: (1) *Absorption*: a photon collides with an atom and an electron jumps to its higher state as it absorbs the photon and its energy; (2) *Spontaneous Emission*: an electron spontaneously falls to its lower energy state and a photon is emitted; and (3*) Stimulated Emission*: an electron is at a high energy state because of a previously absorbed photon, then a stray electron passes the atom, attracts the absorbed photon which is emitted to be absorbed by the stray electron. Spontaneous emission is the basis of virtual photons.

One of the implications of QED is the existence of *virtual particles*. In the 1930s Paul Dirac correctly predicted the existence of the positron, a new particle with the same mass as an electron but with a positive charge and an opposite spin. Dirac's theory implied that virtual electron-positron pairs created by photon decay can add to the universe's mass (hence energy) because the lost photon is massless while both the electron and the positron have mass. But this creation of energy from nothing is a very short-lived phenomenon because the positron is the electron's antimatter—when the two meet they annihilate each other, restoring the photon.

The creation of virtual particles violates the Law of Conservation of Energy, a physical requirement that energy can be transformed from one form to another (heat to light, energy to mass) but it can never be lost or gained by the universe—all the energy that ever existed still exists and will exist forever more, no more, no less. That energy

began in a Big Bang as extremely high frequency radiation, then it cooled over time and spread out into the electromagnetic spectrum as well as combined into matter; it changed its form but not its amount.

In Part 1 we noted that Heisenberg's Uncertainty Principle applies to the conjugates Energy and Time; it says that $\Delta E \Delta t \geq h$: a precise range of time for an event requires a large range of energy. Nuclear reactions occur in very small time intervals, so the Uncertainty Principle says that they must have very large energy intervals.

QED argues that the Law of Conservation of Energy can be violated for very short period periods during which, Feynman showed, $\Delta E \Delta t < h$ can occur, the energy range can temporarily be "impossibly" low. The "impossibly low" energy levels are transferred from the "real" to the "virtual" universe by the creation of virtual particles.

Virtual particles are not copies of their real counterparts. During its brief period of existence a virtual particle is an addition to the universe's energy—it comes out of nothing—and it can take on properties not normally associated with it. For example, a real photon has zero mass, no charge, and no spin. But a virtual photon has mass, charge, and spin.

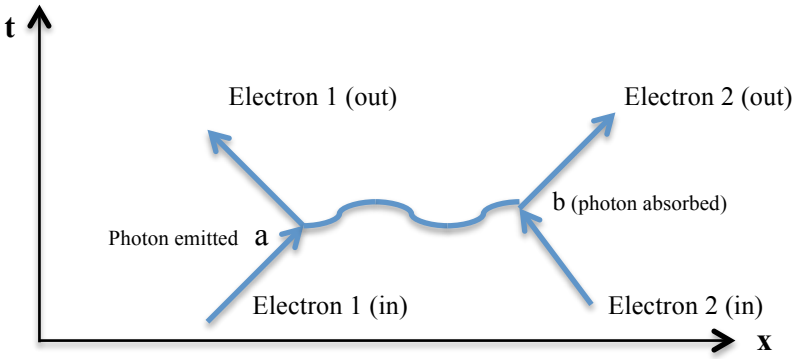
Note that while the lifetime of a single electron-positron pair is extremely brief, if photon decay is happening on a large scale, there will always be virtual mass-energy added to the universe. We return to that issue later,

Feynman introduced a visual method to catalog the interactions that can occur between real particles and virtual particles: the *Feynman Diagram*. In a Feynman diagram an electron is represented by an upward-sloping arrow: the upward direction shows that it moves forward in time; it can take either a leftward of rightward direction. A downward-sloping arrow represents a positron; this does not meant that it travels backward in time, it is simply the symbol for a positron. A photon is represented by a wavy line, with an arrow showing the direction of motion. The point where two particles interact is a *vertex*. In a very complicated example with lots of interactions there are many vertices and the Feynman diagram can be hopelessly confusing. But we will keep it simple.

A Feynman diagram for the electron-photon interactions of spontaneous emission is shown in the figure below. There are three particles: two "real" electrons (which have

mass and a negative charge) and a virtual photon, with mass and no charge. The vertical axis represents time (the up direction is into the future); the horizontal axis measures space. The point where the head of one arrow meets the tail of another is a vertex.

In this example there are only two vertices (points a and b). Electron 1 (the up-slanted arrow) is a real electron that arrives at vertex a, where it spontaneously emits a virtual photon and rebounds to the left as it loses energy to the photon. The photon moves horizontally to point b (it is a fast little bugger, moving at light speed) where it meets with electron 2. Electron 2 absorbs the virtual photon and its energy, bouncing in the opposite direction from its original line of travel.



Spontaneous Photon Emission
Creation of a Virtual Photon

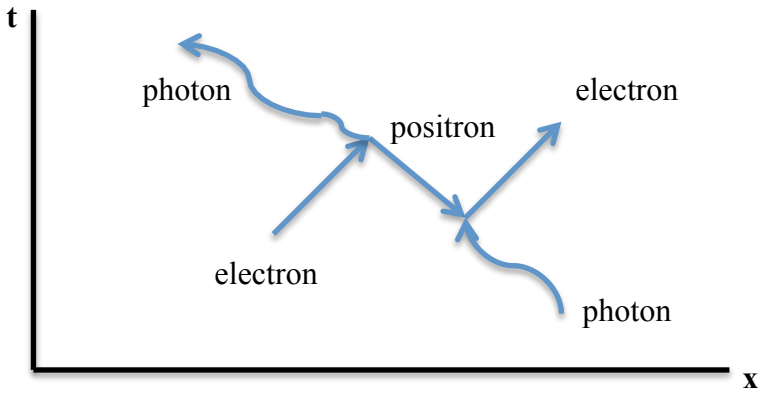
The balance of the universe (the energy of real particles) is restored by the photon's absorption by electron 2, but while the virtual photon exists the universe's real energy increased because the virtual photon has mass. Energy is ultimately conserved, but for a moment it is created.

An example of this process is the spontaneous emission or absorption of a photon when an electron moves to a lower (or higher) orbit around a nucleus. This is essential to Rutherford's shell model of the atom, and to Bohr's discovery that an electron's orbit and its energy are quantized (see Part 1).

Another interaction is shown in the diagram below. An electron emits photons that reverse the electron's charge, converting the electron to a virtual positron; the negative charge is transferred to the virtual photons emitted (real photons have no charge). The virtual positron (positrons are shown as downward-sloping arrows) goes on to

interact with and absorb virtual photons, regaining its negative charge and becoming an electron again.

The creation and almost immediate annihilation of virtual particles is going on throughout "empty" space in vast numbers, creating what have been described as a "sea of virtual particles." So the cosmological effects can be significant.



Creation of a Virtual Positron

Quantum Vacuum Energy and Cosmic Expansion

Einstein's General Theory of Relativity (GTR) explains gravity as the warping of space around areas of mass (galaxies, stars, planets). In its original form, it predicted that the universe would ultimately collapse, but Eisnstein was convinced that the universe would maintain a constant size. He forced this result by inserting a "cosmological constant" in GTR to provide a sufficient amount of negative energy to maintain a static universe.[10] He later regretted this arbitrary introduction of negative energy by an amount that exactly offset the contracting effect of gravity, calling the cosmological constant "the worst blunder of my life."

But after Einstein's 1955 death cosmologists discovered that the universe is actually expanding, and in 1998 they found that it is *expanding at an accelerating rate*;

[10] Positive energy, as in gravity, tends to compress space; negative energy tends to expand space. The "dark energy" that accounts for an extremely high proportion of the universe's energy, is negative energy.

Einstein had been half right—a cosmological constant *was* needed in GTR, but it should have been larger.

Many cosmologists believe that for a very short period after the Big Bang the universe expanded faster than the speed of light, then its rate of expansion began to moderate until about 5 billion years had passed. The rate of expansion began increasing again because with more space there was more *quantum vacuum energy* to accelerate the expansion. If that continues, galaxies will eventually separate from each other at faster than light speed and we will only see our own galaxy.

This *inflationary universe* might be explained by "dark energy" that fills otherwise empty space. The basis of this dark energy is quantum virtual particles created out of nothing. Just as an electron has a minimum energy level so that lower shell electrons can't fall into the nucleus, so every point in space has a minimum energy level called quantum vacuum energy or, more simply, Zero Potential Energy (ZPE). This is measured as ZPE = hf/2 (½ of Planck's constant times the frequency of the energy wave); ZPE is ½ the energy of a photon with frequency f.

As an interesting aside, in 1948 Hendrik Casimir proposed using ZPE as a free source of energy: a *Casimir Engine*. He noted that if the gap between two electrically neutral metal plates is filled by a vacuum, electromagnetic radiation will spontaneously arise between the plates; vibrating at frequencies with integral wavelengths (just as a violin string must vibrate at frequencies with integer or ½-integer wavelengths). The result is that the energy level between the plates is less than the energy level around the plates, leading to attraction between the plates. This is called the *Casimir Effect.*

The movement of the plates generates a small amount of energy that can be captured. But this is no perpetual motion machine: once the plates are at their minimum distance from each other, they have to be pushed apart to start the next cycle—and that requires inputting the energy that has been captured. Thus, a Casimir Engine would use all the energy it produces.

Summary

In Part 1 we reviewed the history of quantum physics and examined some of its strange implications—that all quantum states simultaneously occur until a measurement is taken; that when a measurement is taken, only the measured state exists and the others vanish; that quantum particles seem to revise their histories when their states are measured; that even when totally unobtrusive, the act of measurement doesn't just reveal a result, it *determines* the result; that entangled particles can exhibit "action at a distance," in which they respond instantaneously to a change in the state of their entangled sibling even when separated by great distances. This is the way the subatomic world behaves. We don't know why, but we do know that it is what it is.

Part 2 has addressed the formal foundations of quantum mechanics. A quantum system has a number of basis states that exist simultaneously in a superposition state for which the probability of each basis state can be calculated. The superposition of basis states is the foundation of the probability wave that describes the evolution over time and space of the quantum system. There can be situations in which particles have symmetric states and these particles—called bosons—tend to aggregate together. In other situations particles have antisymmetric states; these particles—called fermions—repel each other. We discussed the Pauli Exclusion Principle that rests on the distinction between symmetric and antisymmetric states and is the basis for the model of the atom and the distinction between particles in the Standard Model.

Some developments in quantum theory were discussed: the concept of entanglement; the theory of Quantum Electrodynamics, with its Sum-Over-Histories method of examining particle interaction; the existence of virtual particles that temporarily violate the Law of Conservation of Energy, and play a role in the "dark energy" that fills empty space and leads the universe to expand eternally.

Parts 1 and 2 provide a summary of quantum theory. In part 3 we will cover applications of quantum theory to computing and to cryptography.

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