Part 1

Foundations of Quantum Mechanics:

History and Interpretation

Peter Fortune

Part 1 of this four part series reviewed the history, development, and interpretation of quantum mechanics. This was done in a nonmathematical fashion appropriate to a general background of the field.

Part 2 reviewed some of the details of quantum theoretical methods. The objective was to lay out the gist of the field with a minimal level of mathematics.

Part 3 reviews issues in Classical and Quantum Information Theory, focusing on Cryptology and Computing

Part 4 is a Technical Appendix

Do not keep saying to yourself, if you can possibly avoid it, 'but how can it be like that?' because you will get 'down the drain,' into a blind alley from which nobody has escaped. Nobody knows how it can be like that.

Richard P. Feynman
The Character of Physical Law (1965)

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Foundations of Quantum Mechanics

History of Quantum Physics

Classical (Newtonian) mechanics addresses the forces and motion of *macro*scopic objects like baseballs, airplanes, and planets. The fundamental assumption of classical mechanics is that the laws of motion are deterministic—if you know the starting conditions (say, initial angle and velocity at which a baseball is thrown) and you know the other forces working on the baseball (say, gravity and wind resistance), then you know precisely the path the baseball takes.

An additional axiom of classical mechanics is the "corpuscular" theory of matter—everything consists of particles, that is, discrete lumps of stuff. Newton thought that even light was corpuscular, composed of what we now call photons (on that point we will see that he was half right).

Quantum mechanics is a horse of a different species. It addresses the motion of *micro*scopic particles, particularly subatomic particles like electrons, protons, photons, muons, tauons, etc, etc. It assumes that the laws of motion are random (an electron could be here, there, or everywhere at once), and it considers its subjects (e.g. electrons, photons, muons) to exhibit duality, that is, the properties of both particles *and* waves.

Wave-Particle Duality

In 1801 Thomas Young first demonstrated *wave-particle duality*. To do this he set up a simple experiment in which a piece of foil with two tiny vertical slits close together was placed in front of a screen that recorded particles of light, similar to a photographic plate. He shined a light through the foil and examined the pattern displayed on the screen. If light was "corpuscular" the pattern would be like Figure 1a: the photons in a light beam would go through each slit like little bullets and, as the light continued to shine, it would build a pattern on the screen of two vertical bars, like Figure 1a.

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But if light behaves as a wave, the pattern would be something like Figure 1b. The waves of light would pass through each slit like water waves through a narrow opening. Once through a slit, the waves would spread out in the process called *diffraction*. When the two waves (one from each slit) intersect there would be *interference*. If the two waves are both at a peak (or a trough) when they intersect, the pattern would show a series of strong dark bars—this is called "constructive interference." But if one wave is at a peak and the other is at a trough when they intersect, there will be no light on the screen—the two waves will cancel each other out in "destructive interference." A series of vertical bars would appear representing the areas of constructive and destructive interference.

Pattern 1a (Particle)   Pattern 1b (Wave)

What did Young see? When one slit was covered and the other left open he found a single vertical bar—the photons were particles. He expected that when both slits were open a pattern like Pattern 1a would emerge: the photon bullets would show as two vertical patterns. But when both slits were open he found a series of light bars with emptiness between each pair like Pattern 1b—now the photons acted like waves showing areas of constructive interference (the black bars) surrounding areas of destructive interference (the blank bars). So, Young concluded, photons are particles *and* waves.

You can imagine the stir this created. Newton's corpuscular theory of light was wrong, or only half right! Furthermore, the behavior of light (particle or wave) depended on the experimental setup: particles with one slit open, waves with two slits open. Thus, physical principles (wave or particle) were not independent of the way they were measured: the experiment determined the result!

And there the matter stood for a very long time. Physics advanced rapidly in the 19th century—James Clerk Maxwell's discovery of the equations describing electromagnetic radiation was a prominent example—but wave-particle duality remained

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on the sidelines. Not until 1900 would quantum theory begin, an understanding of wave–particle duality begin. Some discussion of the characteristics of waves is in order before addressing the development of quantum theory.

Wave Characteristics

Waves are an inherent part of nature. Any medium can generate waves—water, sound, light, electricity, gamma rays—but let's use electromagnetic radiation, of which electricity and light are specific forms, because it plays a large role in quantum theory.

A first characteristic of electromagnetic radiation is its *frequency*, denoted as f. Frequency is the number of complete cycles a wave goes through in a unit of time. For example, in the U.S. electrical power is distributed at a frequency of 60 Hertz, i.e., 60 cycles per second. Power reaching your kitchen toaster completes one wave cycle every $1/60^{th}$ of a second, that is, every .017 seconds. Frequency is very important to the recipient of the wave: your toaster "expects" one cycle every .017 seconds; a mariner is "comfortable" with low frequency water waves and very unhappy with high frequency waves that slap rapidly at his vessel; visible light has different colors at different frequencies (red light has a low frequency, blue light is has a high frequency). Outside the visible range of light, the frequency of radiation goes from extremely low (radio waves) to extremely high (gamma rays).

A second wave characteristic is its *amplitude*, denoted as A. For electromagnetic radiation the average wave value that determines the average brightness (intensity) but the intensity rises to a peak and then declines below the average to a trough. The peak is at +A amplitude, the trough is a –A , and the average is zero.[1] The same definition applies to the amplitude of any wave. For a water wave the amplitude is ½ of the distance from trough to peak: a 10 foot trough-to-peak means an amplitude of +5 feet at the peak, and an amplitude of -5 feet at the trough, with an average amplitude of zero. It has long been known that the intensity (brightness) of light is directly related to the amplitude of its waves, but the relationship is not linear; rather, it is exponential—intensity is equal to the square of the light's amplitude.

1 Light intensity, or brightness is the energy of the light wave per unit of area of the source (a large 100W light bulb is brighter than a small 100W bulb). So the amplitude is directly proportional to the light's energy, and inversely proportional to the light source's area.

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A third characteristic is *wavelength*, denoted as λ. As a water wave advances there is a specific distance between the crests or troughs of the wave; that distance is its wavelength. This is true of all waves with a periodic cycle. Red (low frequency light) has a long wavelength; white light (high frequency) has a short wavelength.

A fourth wave characteristic is its *velocity*, denoted as v. The velocity at which electromagnetic radiation travels is exceedingly high: In a vacuum, all radiation travels at a constant velocity of about 300,000 km/sec, the speed of light. We know that *the velocity at which a wave or a particle travels is the product of the wavelength (λ) and of the wave frequency (f), that is* $v = \lambda f$. So, for example, U.S. electrical power frequency of 60 cycles per second would have a wavelength of 5,000 kilometers if it traveled in a vacuum.

Light Wave Characteristics

- Frequency (f) determines the light's color and energy
- Amplitude (A) determines the light's brightness (intensity)
- Wavelength (λ) is the distance between successive peaks (or troughs)
- Velocity—the speed at which a peak advances in space—is the wave frequency times its wavelength ($v = f\lambda$). In a vacuum, light travels at 300,000km/sec
- Phase is the amount by which one wave is coordinated with another wave of the same wavelength: waves with cycles beginning at the same location are "in phase;" waves that peak when another troughs are "inverted"

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Max Planck and the Beginning of Quantum Mechanics

In 1894 Max Planck, a German physicist, was hired by electric companies to determine how to maximize the brightness (intensity) of light bulbs. He did this by investigating the properties of *blackbody radiation*—energy emissions by a perfect radiator that emits radiation uniformly over its surface. Light bulbs are not blackbodies, but the principles he discovered would be important to their function.

As energy (heat) is added to a blackbody, the emitted radiation increases in frequency from invisibly low frequencies to visible low frequency (red light), up through blue light, white light, and, finally into the invisible ultraviolet range of very high frequencies. Classical theory said that this process was continuous and with sufficient energy added to a blackbody the emitted energy could become infinite. This came to be called *the ultraviolet catastrophe* because the radiation would turn everything to toast. But experiments showed that this was not correct: emitted energy increased with frequency up to a point, but then additional increments in emission frequency actually reduced the intensity of the light emitted. This was a blow to classical theory.

In 1900 Planck reported a startling results: energy is emitted in specific discrete amounts called quanta. A blackbody could emit one quanta of energy, two quantas, three quantas, and so on, but it could not emit 1.75 quanta or 20.22 quanta. This was in stark contrast to classical theory that allowed energy emissions in any continuous amount. Planck argued that the reason there was no ultraviolet catastrophe—that emitted energy actually fell at very high frequencies—was that as energy was added to a blackbody and the frequency of emitted light increased, the energy required to emit a quantum of energy also increased and at a finite energy level you couldn't generate the energy required to go higher. You had to push harder and harder on the accelerator to get any increase in energy emission.

Planck's Law

The energy emitted by a blackbody is in discrete quanta that depend on the frequency of the emission. The amount of one quantum of energy is

$$E = hf$$

(h = Planck's constant = 6.66×10^{-33} joule-seconds; f = frequency)

Wave energy is directly proportional to the emitted wave frequency. The constant of proportionality is Planck's Constant (denoted as h), which appears in every law of quantum theory. It defines the minimum amount of that characteristic, that is, the size of a quantum. Planck's constant also defines the quantum of length and of time. All "stuff" comes in discrete units even though those units are too small for us to notice.

In 1905 Einstein used Planck's Law to explain the *photoelectric effect* (he received the 1921 Nobel Prize for this—not for his more important contributions--special relativity and general relativity). When light is directed at a metal plate electrons can be "kicked out" of the plate's atoms; Einstein called these *photoelectron*s. Classical theory had at that time accepted the view that light is a wave, and it predicted that the number of electrons dislodged would be related to the intensity of the light, that is, to the frequency of the light's wave. But experiments showed that the number of electrons scattered from the metal depended on the frequency of the light wave, not on its intensity. Einstein explained this as a consequence of Planck's Law: Light is particles called *photons* and the energy of a photon is directly related to its frequency. So higher frequency light carries more energy and kicks out more electrons.

Jumping ahead, in 1924 Louis de Broglie reported a startling result in his doctoral dissertation. *All* matter is wave-like, whether it is electrons, atoms, molecules, baseballs, or railroad locomotives. Duality does not apply to light alone—it applies to everything. Even a baseball acts as a wave, though the object has such a short wavelength that we see only the thing, not the wave.

What de Broglie found was an equation for the *wavelength of matter*. If you recall your high school physics, momentum (designated by p) is equal to mass (m) times velocity (v), that is, p = mv. A bullet can hit its target with the same momentum as a locomotive hitting a brick wall if the bullet (with low mass) is traveling fast enough and the locomotive (with high mass) is traveling slow enough.

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de Broglie's Law

The momentum of a particle is inversely related to the particle's wavelength. As a result:

$$p = h/\lambda$$

or $\lambda = h/mv$

or $v = h/m\lambda$

or $m = h/v\lambda$

(h = Planck's constant = 6.66x10^{-33} joule-seconds; λ = wavelength of particle)

So what de Broglie showed is that momentum (hence velocity) is inversely proportional to wavelength. Note the presence of Planck's Constant.

Nils Bohr and Ernest Rutherford's Model of Atomic Structure

In 1911 Ernest Rutherford bombarded gold foil with electrons. He postulated that the gold atoms in the foil would have large spaces between each atom because electrons have a negative charge and would repel one another, increasing the space between atoms. Therefore, he argued, most electrons in the beam would pass directly through the foil, experiencing only a slight change in direction as an incoming electron interacted with the magnetic field of a gold atom's electrons.

But what he found was quite different: the electrons in the beam scattered wildly in many directions, some coming straight back at the beam gun. This *scattering* revealed something deep about the atom. Rutherford postulated the *planetary model of the atom*: atoms consist of a large and heavy nucleus, and collisions between the beam's electrons and gold nuclei were creating the scatter pattern, just as a stream of bullets shot into a field of rocks will ricochet in many directions. That indicated that there was a heavy body in the atom, a nucleus. He also knew that an atom had a neutral charge and that electrons are negatively charged, so the nucleus must be positively charged. Thus, the

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nucleus repelled the electrons but provided a gravitational attraction that induced the electrons to orbit the nucleus like planets around the sun.

But Rutherford's model of electrons as particles orbiting a large nucleus was subject to a fatal problem. If true, classical theory predicted that an atom would be unstable—it would quickly destroy itself. Maxwell's equations for electromagnetic fields showed that the motion of electrons in orbit creates a magnetic field that tugs at the electrons, slowing them down and reducing their energy. As the energy of orbiting electrons declines, electrons fall into lower and lower orbits. Eventually the lowest electrons collide with the nucleus and the atom disintegrates. Thus, the planetary model implied that atoms were only ephemeral particles with an extremely short half-life.

In 1913 Nils Bohr discovered the reason that atoms don't disintegrate. He found that the radius of an electron's orbit of an electron changes only in discrete amounts, in contrast to the classical theory that said that any change in energy would change the radius. Each orbit is a certain distance from the nucleus, and changes in energy can induce shifts from one orbit to a higher orbit (if you add enough energy) or to a lower orbit (if you sufficiently reduce energy). There is no orbit allowed between any two allowed orbits and—crucial to the stability of the atom—there is a minimum allowed orbit radius and, consequently, a minimum allowed energy level for the atom's electrons. An atom's electrons cannot fall into the nucleus because the lowest orbit of electrons won't allow it.

Bohr also concluded that electron waves must be *standing waves*, waves that appear fixed in place and simply oscillate vertically up to a maximum positive amplitude and down to a minimum negative amplitude. Standing waves are often found when a wave of a specific frequency moves in one direction while a wave of exactly the same frequency moves in the opposite direction, as when a water wave hits a seawall, rebounds, then interacts with incoming waves. If the two waves are out of phase by exactly ½ wavelength, the resulting "wave"" will be flat; if the two waves are exactly in phase, there will be constructive interference and the resulting wave will move vertically with a doubled amplitude; that is a standing wave. The waves interfere in a way that makes them appear to be just moving up and down instead of advancing.

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Consider plucking a violin string. The string is attached to the violin body at both ends so it cannot vibrate at its ends; all vibration must be between the ends. If the string is plucked a wave travels down the string to the other end. Then it echoes toward its source with the same frequency and, initially, almost the same amplitude. A standing wave is created that dies down ("dampens") as the amplitude of the echoes decreases due to the energy emitted as sound or absorbed by vibrations in the violin body.

The wavelength of a standing wave can occur only in discrete quantities (quanta again): the length of the string allows either ½ mavelength, 1 wavelength, 1½ wavelengths, 2 wavelengths, 2½ wavelengths, or so on. Intermediate wavelengths generate distorted tones. If the violinist wants to change the frequency of vibration on a string, she uses the frets to shorten or lengthen the string to allow precisely the allowed wavelength.

Electron waves must follow the same rules because in any orbit both ends of the wave are attached to the average orbit radius. As energy is added to an electron, pushing it to a higher orbit, the orbit radius increases and with it the length of the standing wave increases, but the radius increase must be such that the new standing wave has the property that the radius is ½ wavelength, 1 wavelength, 1½ wavelengths, and so on. De Broglie had found the reason that orbits are quantized: only radius changes that allow electrons to orbit in standing waves are allowed, so only discrete changes in energy can alter an electron's orbit.

Quantum Theory in the 1920's

The 1920's was a period of major advances in quantum theory. Planck's explanation of the stability of the atom had introduced the idea that energy is released or absorbed by an atom's electrons in discrete quanta defined by the energy content of a photon. This was the seminal idea in quantum mechanics, but it left a question hanging: a tenet of scientific advance is that new theories should be compatible with the old theory under certain circumstances. For example, Special Relativity is consistent with classical physics for relative motions considerable slower than the speed of light; this observation boosted support for Special Relativity.

But are classical mechanics and quantum mechanics compatible under any conditions, or does nature have to be described by two separate laws of physics?

In 1920 Bohr proposed the *Correspondence Principle* and put quantum theory on a firmer footing. Classical theory and quantum theory are not separate and competing theories. Rather, they each apply to particles of different sizes: classical theory views things as particles of "large" size; quantum theory sees things as waves or particles (depending on the circumstances) and applies to subatomic sizes. But what is that critical size? Bohr calculated the critical object size, calling it the *Correspondence Limit.* Thus, quantum theory embraced classical theory instead of conflicting with it. Size matters!

In 1926 Erwin Schrodinger made a major discovery. If an electron behaves as a wave, he realized, it must obey equations that would describe any wave. There must be a "wave function" that describes the amplitude and frequency of the electron's wave at each point in time and space. Schrodinger discovered the exact mathematical form of that wave equation. *Schrodinger's wave function,* denoted by Ψ, is a second-order partial differential equation. One form of it is given below (there will *not* be a test). It is a second-order partial differential equation (got that?)

Schroedinger's Wave Function

$$d^2\Psi(x)/dx^2 = -(2m/h)E\Psi(x)$$

(Ψ = wave amplitude, x = distance in orbit, E = particle energy)

What does the wave actually represent? In 1926, right on the heels of Schrodinger's insight, Max Born intuited that just as classical theory said that intensity of light is the square of its wave's amplitude, so the Schrodinger wave is a *probability wave* that measures the probability that the particle is in a specific quantum state.[2] Suppose that state is its spatial position. We can't say exactly where the electron is; we can only state the probability that it is at a specific point. So if $\Psi(x)$ is the wave's amplitude at point x, $|\Psi(x)|^2$ is the probability of the electron being at that specific position. This is known as *Born's Rule*.

2 The example in the box assumes that the location defines the only state, but there can be several state characteristics—location, time, energy, etc—in the Schrodinger wave function.

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Born's Rule

$$P(x) = |\Psi(x)|^2$$

P(x) = Probability that the particle is located at point x

This was a disturbing concept. An electron might be a "thing" but it had no known location. All we can say is that its orbit must allow a standing probability wave (Bohr's insight), and that the square of the wave's amplitude is the *probability* that it is at a specific point in its orbit. Nature at the subatomic level is inherently random, filled with possibilities but with no certainties.

Think back on Young's two-slit experiment. In classical physics, a bullet (a photon) is shot and passes through one or the other slit. Because the bullet can't pass through both slits, the probability that it reaches the screen is the probability that it passes through the first slit plus the probability that it passes through the second slit. But in quantum theory what is being transmitted by the light beam is a probability wave, not a bullet, so—like any wave—it can pass through both slits. When it emerges on the other side it can have destructive interference, in which case it does not reach the screen, or it can have constructive interference, in which case it reaches the screen twice (once from each slit).

In 1927 Werner Heisenberg created another revolution in physics. Heisenberg found that if two characteristics of a particle are conjugates (that is, have related characteristics, i.e., quantum states) then more precise knowledge of one characteristic *necessarily* implies less precise knowledge of the other. Consider a particle's momentum and its position; these are conjugates characteristics because both depend on the particle's velocity. Classical theory said that in principle you can simultaneously and precisely measure both the momentum and position of a particle, just as you can simultaneously and precisely measure the momentum and position of a thrown baseball. But not so!

The culmination of this work was summarized as *Heisenberg's Uncertainty Principle*.

Heisenberg's Uncertainty Principle

The more precisely a particle's characteristic is measured, the less precisely can the conjugate characteristic be measured

• *If p is a particles's momentum and x is its position, then*

$$\Delta p \Delta x \geq \hbar$$

• *If E is a particles's energy and t the time it is measured, then*

$$\Delta E \Delta t \geq \hbar$$

($\hbar = h/2\pi$ is Planck's Reduced Constant)

Heisenberg's Uncertainty Principle says that there are fundamental and irreducible uncertainties in particle states, and that the measure of that uncertainty is Planck's Reduced Constant: the product of the two ranges can be no less than $\hbar$, so if position is measured with very high precision, momentum (velocity) must be measured with very low precision. This extends to all conjugates, such as energy and time: the better you can determine the time at which a particle takes on a certain state, the less precise you can be about the particle's energy.

It is important to realize that this uncertainty is not due to limitations in experimental design. For example, it is not because of the *observer effect* that says that since you can only measure position by shining light on the particle, the photons in the light will disturb the particle, changing its velocity and, therefore, its momentum. The uncertainty is an irreducible fact of nature. It is the way God designed the subatomic world. This is not to say that there is no observer effect, only that even if there weren't, there would be the tradeoff in precision of measurement of conjugates.

Heisenberg's Uncertainty Principle has been supported by every experiment. For example, studies of wave diffraction using all kinds of waves have supported it. When

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water wave passes through a narrow opening (like through a gap in a breakwater) it spreads out in a particular way: the wavelength (distance between peaks) increases, and the resulting wave takes on many angles as it continues on its way; we see this as the arc-shaped form of a water wave as it exits the opening. That phenomenon is due to the friction of the opening's edge on the entering wave: the portion of the wave passing through the center of the opening has minimal friction, but as you look toward the edges of the opening there is increased friction and the incoming wave is slowed by the edges of the opening, changing its direction of travel.

Suppose that you want to measure the new wavelength. Classical theory will give a precise answer: if W is the width of the opening, and λ_0 is the incoming wavelength, the outgoing wave has a wavelength of λ_0/W: an increase in the wavelength can be created by narrowing the width or by increasing the length of the entering wave. Similarly, the angle of exit of the new wave at any distance across the opening will be larger the narrower the opening. The exit angle and the new wavelength are conjugates. Numerous experiments on particle scattering have confirmed that more precise measurement of one (exit angle) implies less precise measurement of the other (wavelength).

Foundation Concepts of Quantum Theory

(1) Duality: subatomic particles behave like waves *and* like particles, depending on the specific experiment

(2) Physical phenomenon (energy, orbit radius, etc.) are quantized, that is, it comes in discrete packets (*quanta*)

(3) A particle's characteristics (position, momentum, etc.) are in quantum states whose evolution is described by a probability wave

(4) Until a measurement is made a particle exists in all quantum states; when a measurement is made the other states vanish; the probability wave collapses.

(5) The quantum state of a system can never be precisely known; all we can know is its probability of occurrence

(6) The probability of a specific quantum state is the square of the probability wave's amplitude

(7) Quantum mechanics can not be explained but it can be understood through its mathematics,

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The Interpretation of Quantum Theory

Are you confused? You should be. There are two kinds of people who don't understand quantum mechanics—the new physics student, and everyone else. Richard Feynman, the great 20th century physicist who discovered quantum electrodynamics, said that he didn't really understand the meaning of quantum mechanics; he only understood the theory and he knew that it worked.

The difficulty in understanding quantum theory is underscored by a running debate between Albert Einstein and Nils Bohr. As we have seen, Bohr was the first to propose that nature comes in packets called quanta. Einstein built on this new quantum theory when he explained the photoelectric effect, for which he won the Nobel Prize. But Einstein never really bought quantum theory as a complete description of subatomic particles; he thought something was missing.

Einstein vs. Bohr: The Complementarity Debate

Bohr referred to conjugate variables as complementary. Recognizing this, the debate between Einstein and Bohr about the fundamental meaning of quantum mechanics, called the *Complementarity Debate,* was centered on the Uncertainty Principle. It began in 1927 and continued well into the 1930s, actually continuing until Einstein's death in 1955.

Bohr advanced the *Complementarity Principle*, arguing that there is no equipment or experimental design that could overcome the Heisenberg Uncertainty Principle—it is an inherent part of nature when particles exhibit duality (e.g. wave-particle)—conjugate characteristics (also called *complementary characteristics*) could not both be measured with precision. This, he thought, was an essential component of quantum theory.

Einstein never accepted the randomness that quantum mechanics implied; hewas famously paraphrased as saying, "God doesn't play with dice." (Bohr responded, "Stop telling God what to do!) At heart, Einstein was a Newtonian in his adherence to determinism, even though his Special Relativity destroyed Newton's theory of time and space. He believed that with proper understanding of nature and with proper equipment, both position and momentum could be precisely measure. Any remaining uncertainty

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would be due to the observer effect—the act of measurement disturbs the particle being measured.

At the heart of the debate was a deep difference in the philosophy of science. Einstein was a *philosophical realist* who believed that nature "made sense" and that if we couldn't describe that sense, our understanding was incomplete. His paradoxes arose from a situation that didn't make sense to him. Bohr was not wedded to the idea that things were explicable in every day language—that would be like using classical theory to explain quantum mechanics. So he was willing to "follow the math" wherever it led.

As a philosophical realist, Einstein believed that what we see as Heisenbergian uncertainty is really a measure of our ignorance about the true laws of physics, particularly the effect of excluded variables. The simultaneous measurement of both momentum and position looks imprecise because it depends on things that affect both but which we have neglected to consider. For example, if the intensity of the sun's cosmic rays affect a particles position and momentum, and if we only included that in our measurements, the uncertainty would disappear. In essence, he believed that quantum theory is incomplete and contained inconsistencies hidden from us.

The Complementarity Debate consisted of a running dialogue between Einstein and Bohr. Einstein would propose a quantum theoretical "thought experiment" that created a paradox and revealed inconsistencies in quantum theory. Bohr would respond, sometimes after protracted thought, and solve the "paradox" using quantum theory. Einstein would concede that Bohr was correct on that matter—and then propose another paradox. This went on for years. Finally Einstein conceded that none of his proposed inconsistencies held water and that he was fresh out of new paradoxes. But he still adhered to determinism; he thought that he just hadn't asked the right questions.

One of the most famous of these paradoxes was the EPR paradox, named after a paper published in 1935 by Einstein, Boris Podolsky and Nathan Rosen. This paradox rests on the then-new concept of *entangled particles*, two (or more) particles that shared a quantum state and, therefore, had related characteristics even if they were widely separated in space. Entangled particles share the wave function so that quantum theory can predict the state of one particle simply by measuring the state of the other. An

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extreme example is if there are both a cat and a dog in Schrodinger's Box: the fate of one is perfectly correlated with the fate of the other.

Conjugate variables apply to the characteristics of a single variable: position and momentum of an electron. But entanglement applies to multiple particles. An example of entanglement is the *spin* of a positron and its related electron.[4] Those two particles are created when a photon decays into a positron and an electron. The probability wave of the photon is transformed to a probability wave of the two particles. The positron must have a spin direction opposite that of the electron: if, at their creation the spin of one is "up" the spin of the other is "down."

Suppose, EPR say, that a positron and an electron are shot off in opposite directions until one reaches observer A and the other reaches observer B. To make it dramatic, suppose A and B are one light-second (300,000 km) apart. When the particles start off their spins are unknown—In fact, each has *both* "up" spin and "down" spin because the quantum state is a superposition of all possible states, meaning that the state of the two-particle system is an "average" of up spin and down spin, just as Schrodinger's Cat is an average of dead and alive. So the spin that would occur if the particle is measured is random.

Now suppose that when A and B receive their particles, A measures his particle's spin and finds "up." Quantum theory says that the probability wave for B's particle immediately collapses to "down" because opposite spins are required. So B *immediately* records a down spin. A and B find opposite spins even though they are far apart. This is not a fluke, it happens every time: A find up, B finds down; A finds down, B finds up.

The puzzle is, how did B's particle "know" that an up spin was found for A's particle? Einstein believed in the *locality* of all quantum interactions, that is, they can only work by traveling through space at a speed no greater than the speed of light—information takes time to flow to the receiver. *Non-locality* means that the information about spin travels faster than light, perhaps instantaneously; this violates special relativity and is, therefore, impossible. But quantum theory argues that interactions can be *non-*

4 Spin will be discussed in Part II.

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local, even occurring instantaneously. Somehow, even at great distances the measurement of spin direction of one particle determines the spin direction of the other.

Einstein described non-locality as "spooky action at a distance. If it takes a finite time for information to travel, how can A's measurement of the spin as "up" immediately mean that B measures a "down" spin? How can measurement on one particle instantaneously cause the other particle's wave function to collapse? The EPR paper argued that quantum theory was incomplete and that there must be "hidden variables" that determine the correlation between entangled particles. Take that, Dr. Bohr!

This was one of the most difficult paradoxes that Bohr faced. It was a "thought experiment" long before equipment was good enough to conduct an actual experiment, so he couldn't just go to the lab and perform an experiment. Of course, even if he could, the experiment wouldn't be definitive—no matter how many times an experiment is performed with the same result, the next time it *could* have a different result.

Bohr's response won the day. He argued that while it was true that no "mechanical" transmission could occur faster than light speed, so the information of a measured state at one variable required time to transmit to the other variable (in our example, one second is required) there is nothing in quantum theory that requires a probability wave to limit itself to a small area: the EPR paradox was not a paradox, it just was a failure to understand that the same probability wave would describe both particles. Slam dunk!

Who won the debate? Clearly, Bohr was the victor because he could always use quantum theory to explain a paradox to Einstein's satisfaction. But who was right? The jury is still out. However, advances in both theory and experimental physics still give the edge to Bohr.

In 1964 John Bell produced a paper describing explicit criteria for determining whether a local form of Einstein's hidden variables theory—one obeying special relativity—is correct. For a *local hidden variables* theory hidden variables exist and would allow all measurements with high precision, but the speed limit in special relativity would be obeyed. These criteria were called *Bell's Inequalities*. Essentially, they said that if the correlations between quantum states exceeded the inequalities, a local hidden variables theory could not be valid.

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Bell's paper made two major contributions. First, it found that whether interactions satisfied local hidden variables depended on certain parameters of the quantum system—it was a matter for experimental determination, not for theoretical solution. Second it described the precise conditions under which an experiment could conclude in favor of Einstein. This allowed experiments to determine if results supported locality (Einstein) or non-locality (Bohr). Technological improvements have allowed a number of those experiments to be conducted.

There is continuing debate about Bell's Theorem and the experiments that have been conducted. But at this date non-locality has been supported: a quantum theory with local hidden variables is not valid.

So the Complementarity Debate ended in a qualified victory for complementarity and action at a distance (non-locality). It was a victory for quantum theory, not just for Nils Bohr, because the Einstein-Bohr dialogue brought fundamental issues to the forefront and helped everyone to a better understanding of the interpretation of quantum theory—not to an understanding of *why* it worked, but to an acceptance of the idea that it *did* work.

Heisenberg and Bohr: The Copenhagen Interpretation

Over the period 1924-1927 Bohr, Heisenberg and others worked to develop a quantum canon, that is, an "official" interpretation of quantum theory. The result was the *Copenhagen Interpretation*, so named because Bohr was Danish. This had a number of tenets: (1) all matter behaves as both a particle and as a wave; the particle has energy, and the wave is a description of the transmission of that energy; (2) all we can know about a particle's state (position, motion, energy) is the probability that it has a certain state, described by Schrodinger's probability wave function; we can never know what is the exact state; (3) A particle can be viewed as having all possible states, each with its own probability, but when the state is measured in an experiment all the other states disappear. This is referred to as the "collapse of the wave function;" (4) No experiment can measure conjugate states of a particle with certainty; more precise measurement of one characteristic (momentum) implies less precise measurement of other characteristics (position).

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Perhaps the most contentious of these tenets is the third. It says that all possible states exist simultaneously until the state is measured and only one state is found: *a particle is everywhere until it is found to be somewhere*! A simple example is when a photon is emitted at point A and aimed at point B, where a detector says that it has arrived. If the photon were a bullet, we would say that it traveled in a parabolic arc from A to B. In the quantum world we might expect the same of a photon. But the Copenhagen Interpretation says that the photon takes every possible path—a straight line, a route directly away from the target with a 180 turn, a route around the moon and back to B—*all* possible paths! Each of these routes has a measurable probability: the route around the moon is highly improbable; a straight line to B is highly likely. It is only when we subject the system to a measurement that the answer become specific and all other routes disappear. If we ran another experiment under exactly the same conditions the measured route would be different because it is probabilistic, not deterministic. Repeated experiments can uncover the probabilities attached to different routes, but it can never tell you which route will be taken.

We will revisit this point when we discuss experiments using an interferometer. But Schrodinger gave a tongue-in-cheek example that has become part of the popular dialogue. It is called *Schrodinger's Cat*. Suppose there is a closed box with a cat in it. Also inside the box is a mechanism that releases a fatal gas after a randomly determined time has elapsed: it might activate ten seconds or ten years after the cat is enclosed; all we know is the probability attached to each time-to-cat-death.

You have closed the box and gone on vacation. Is the cat dead or alive when you return? Now, you firmly believe that the cat is either dead or alive—it can't be both—and that when you open the box you just see the actual state. But according to the Copenhagen Interpretation of quantum theory, before you open the box the cat is both dead *and* alive! The quantum state of the box is defined as a *superposition* of the individual states, that is, it is the sum of each possible state multiplied by the probability of that state. Thus, if $|alive\rangle$ is the state "alive," $|dead\rangle$ is the state "dead," and P is the probability of state $\bullet|alive\rangle$, then the state of the cat, i.e., $|cat\rangle$ is[5]

$$|cat\rangle = P(alive) \bullet |alive\rangle + [1 - P(dead)] \bullet |dead\rangle$$

5 The notation used is Dirac's "bra-ket" notation. A "ket" for a quantum state S is denoted as

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Suppose that just before you open the box the probability of the gas having been released is 0.25, so P = .75. Suppose also that we assign values to the individual states, so |alive > = 1 and |dead > = 0. The cat's quantum state before opening the box is 0.75. It is both dead *and* alive, but only 75% alive. In quantum physics you *can* be 50 percent pregnant!

Opening the box is an act of measuring the cat's state. Suppose that when you open it the cat is alive. Now the cat's state is |cat > 1. The wave function has collapsed to one value and the other possibility (dead) is gone. *Measurement causes the wave function to collapse*! Note that in the Copenhagen Interpretation it is not simply that now you see the cat's true state. There is no single true state and by opening the box you have forced the quantum system to select a state. The measurement determines the result!

The Copenhagen Interpretation is not the only interpretation of quantum theory. Another is the *Parallel Universe Interpretation*. According to this, when you measure a quantum state the unobserved states don't disappear—they remain valid but they exist in different universes. For example, before you open the cat's box both states "alive" and "dead" exist simultaneously, but if you open it and find the cat alive there is a branching of universes: you are in a universe with the cat alive, but now there is another universe with the cat dead. Where all those other universes are is a great mystery. Perhaps that is why real estate prices were so high a few years ago.

Does that help you understanding? Of course not! Quantum theory works but it makes no sense.

|S >. The ket is the values of energy, position, velocity, etc. that describe the quantum state. It could be a single value , such as "(alive," or "dead") or it could contain several values; if there are several values, the ket is a vector. The notation P( ) represents probability, and • is a multiplication sign.

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Demonstrating Quantum Theory

Recall Young's two-slit experiment back in 1801? Young's device consisted of a beam of light, a piece of foil with two slits, and a viewing screen that recorded the impacts of photons. A modern analogue is an interferometer. We will use an imaginary *interferometer* to tease out some of the strange properties of subatomic particles that were reported in the previous sections.

The Mach-Zehnder Interferometer

First we should consider the properties of light hitting mirrors. Light striking a mirror can be *reflecte*d at the angle of the mirror, as when a mirror is used to see around corners, or it can be *refracted*, that is, pass directly through the mirror and continue on in the same direction at which it entered (though at a different angle). The light wave might also experience a *phase shift*—it exits the mirror at a different part of its cycle than that at which it entered the mirror. For example, a ½ wavelength phase shift (which we will assume) will invert the wave so that a photon entering the mirror at a point in its cycle with, say, amplitude +A, will exit the mirror with amplitude –A. If a wave and its inverted wave are combined, the result is *destructive interference*: the combination of amplitude +A and amplitude –A generates a flat wave with no amplitude; the wave is cancelled. However, if a wave is combined with another wave of exactly the same form, the result is *constructive interference*: a +A amplitude added to a +A amplitude with no phase shift gives a wave with +2A amplitude. We only need to spend some time on the water to appreciate this truth.

The nature of the mirrors makes a difference to the results. Mirrors with both surfaces made of the same material (say, glass of a specific quality) are often called *unsilvered mirrors*. Mirrors with different materials at the front and back are often called *half-silvered mirrors*. Half-silvered mirrors might be made with glass on one side and glass coated with powdered aluminum (the silvering agent) on the other side. Half-silvered mirrors are a form of one-way glass (familiar from crime shows) used as *beam splitters* in interferometers. Their job is to take an entering light beam and to reflect half of the photons and refract the other half, thus creating two separate light

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beams, or to take two separate light beams and combine them into one beam with different characteristics.

Unsilvered mirrors are simple: they reflect *all* light and they *always* shift the phase by ½ wavelength. Half-silvered mirrors act just like unsilvered mirrors for reflected light: all reflected light has its phase shifted by ½ wavelength. But the refracted light from a half-silvered mirror has a phase shift only if the exit surface has a higher index of refraction than the entry surface; if the exit *index of refraction* is lower there is no phase shift for refracted light. Because glass has a higher index of refraction than powdered aluminum, a phase shift occurs only if the beam enters the silvered side and is refracted through the glass side. These optical properties are the basis for interferometers.[6]

Interferometers come in a variety of flavors depending on their job description. We will assume a Mach-Zehnder Interferometer with the design shown in Appendix, Figure 1 on page 30.

In Figure 1 the blue mirrors are unsilvered glass and mirrors #3 and #4 are half-silvered with glass on the northwest (blue) side and silvering on the southeast (silver) side. Light from a photon gun at the lower left is beamed at mirror #1. There are two paths that a photon can take. The Upper Path (denoted in later text as UP), shown in red, has light reflecting from mirror #1 to mirror #3; it is inverted from its original form. At mirror #3 it is inverted again and reflected to mirror #4. The Lower Path (denoted in later text as LP), shown in black, has refracted light from mirror #1 going to mirror #2 without a phase shift, then reflecting to mirror #4 with an inversion. So UP light arrives at mirror #4 in its original phase (after two inversions), and LP light arrives at mirror #4 inverted.

What happens when the photons at mirror #4 are reflected or refracted to the detectors B and G? That depends on the experimental design, as we see in the next section.

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6 This is oversimplified. Proper choice of materials allow a beam splitter to reflect any proportion of the incident light, and the phase shift need not be ½ wavelength—again, the materials determine the amount of phase shift. But we assume powdered aluminum as the silvering agent, in which case phase shifts are always ½ wavelength and half of the incident light is reflected.

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Interferometer Experiments

We start with the simplest experiment: a One-Path experiment in which the photons of a light beam can only take the Upper Path. This is shown as Figure 2 in the Appendix. To implement this, we replace the half-silvered mirror #1 with an unsilvered mirror that reflects and inverts all light waves. Now all photons emitted by the photon gun are reflected to mirror #3 with an inverted wave; no photons can go to mirror #2. At mirror #3 they are reflected to mirror #4 with a second inversion; they arrive at mirror #4, the second beam splitter, with unchanged phase. At mirror #4 half of the protons are reflected to G with an inverted wave, the other half are refracted to B in the original form. Both B and G click 50 percent of the time because all photons arrive with nonzero amplitude. The bottom line is that all the emitted photons are accounted for and the photons act as particles. This experiment is equivalent to Young's 1801 experiment with one of the two slits blocked.

That was easy! Now let's do a Two-Path Experiment where both paths are open (see Figure 3 in the Appendix). We restore the half-silvered mirror to slot #1 so now the beam is split at mirror #1. We would expect that the Two-Path Experiment would be like running the One-Path Experiment twice: first with the LP closed, then with the UP closed. Thus, G *should* detect 50 percent of the photons, B *should* detect the other 50 percent, and all emitted photons *should* be accounted for.

But Man plans and God laughs! The result is very different from our expectations. Now G *never* clicks because the waves it receives cancel each other—there is destructive interference as shown by the red UP photons arriving at G inverted while the black LP photons arrive unchanged. The other 50 percent of photons do reach B in phase, so B clicks twice as loud. The Two-Path Experiment shows that photons are both waves (because of the interference at G) and particles (because of the clicks at B). But only half of the photons are detected. This replicate's Young's simple two-slit experiment.

For a third experiment we do a "Which-Path" Experiment in which we detect the path that a photon is actually taking (Appendix, Figure 4). Now things get *very* strange. Everything is the same as in the Two-Path Experiment with one benign change that would seem to make no difference: we insert a nonintrusive pass-through detector on the

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UP between mirror #1 and mirror #3. Being non-intrusive, this detector does not affect the characteristics of a photon reaching it; being pass-through, it allows the photons to pass through to mirror #3. All this new detector does is tell the observer which path the photon took: if the detector clicks, the photon took the UP because that's where the detector is; if the detector is silent, the photon took the LP.

One would expect that introducing this benign detector would have no effect on the outcome: the detector changes nothing, it just gives us "which path" information. So the results should be the same as the Two-Path Experiment. But the results reported in Appendix Figure 4, are astounding: *they are identical to the One-Path Experiment* where half the photons are sensed by each detector as particles and no wave characteristics are seen! The wave characteristics of photons have disappeared, all photons act as particles even though they can take both paths, and half of the photons are missing. The mere detection of the path taken precludes the other path from *ever* having been taken.

How can this happen? How can the destructive interference arising when photons can takeboth paths disappear simply because the photon's path can be detected. Why do photons act as waves without detection and as particles with detection? This is a mystery wrapped in an enigma inside a riddle.

There is no answer—we can't talk to photons to find what they do or don't know. But this strange property is addressed by the *third tenet of the Copenhagen Interpretation*: a photon—or any subatomic particle—takes all possible paths simultaneously, the probability of each path being determined by the Schrodinger's probability wave function. It is as if a particle clones itself at mirror #1, going along both paths until it is called back when the observer knows that it is actually on the other path. Photons act like both a wave and a particle (a la the Two-Path Experiment) *unless the observer knows that it is a particle*. At that moment the wave function collapses so that the system acts as if only one path is open. *This is Schrodinger's Cat, both alive and dead until we measure its pulse.*

An important implication of the Which-Path Experiment is that measurement of the quantum system (the interferometer) not only *detects* the result, it *determines* the result. *In a very important sense, everything happens until a measurement is taken!*

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Do you find this mystifying? Just wait a second! In 1978 John Wheeler proposed what he called a delayed-choice experiment. In this experiment, the observer has a choice about whether or not to detect the path taken *after* the path has been taken but before final detection at B or G. In Figure 5 we have inserted two detectors, shown by the oblong blue lenses just before mirror #4. The observer can decide whether to turn those detectors on after the photon has done all its work except the final detection. Surely the result is already baked in the pie by this time: the photon has taken either the Upper Path or the Lower Path and it is too late for the result to be affected by the observer's decision whether or not to activate the lens-like detectors. So the result should be like the Two-Path Experiment without detection, right? Wrong! The result is the same as the Which-Path experiment with automatic detection (Figure 4): if the observer decides to activate the new detectors, the photon acts like a particle and is detected at either G or B with a click; if he does not activate the detectors, then the photon acts like a wave and interference occurs. Wheeler's thought experiment has been verified in the lab. The observer determines the result!

Summary

Quantum mechanics is a theory of the forces and motions of subatomic particles. For large objects, the rules of classical mechanics attributable to Isaac Newton work just fine; but for subatomic objects, classical mechanics fails to accurately predict force and motion. This is very similar to Einstein's Special Relativity, which showed that classical rules work extremely well in the measurement of time and space for two observers moving slowly relative to each other, but that at relative velocities near light speed, time and space measurements differ considerably for the two observers. Special Relativity emphasizes relative speeds; Quantum Mechanics emphasizes relative sizes. But both tell us that classical laws of physics do not universally apply.

The development of quantum mechanics began with Nils Bohr's work on blackbody radiation, work that showed that energy is emitted in discrete amounts called quanta, and that most other physical concepts are quantized. Bohr's work was instrumental in explaining the photoelectric effect and the stability of the atom, among other mysteries. Louis de Broglie advanced quantum theory by showing that all matter is both particle and wave, and by investigating the wave characteristics of particles. Schrodinger and Born further advanced the field: Schrodinger by defining the equations that describe the probability wave of a particle (the Schrodinger Wave Function); Born by identifying the probability that a particle will be in a specific state. Werner Heisenberg introduced the Uncertainty Principle that says that at the subatomic level nature can not be measured with high precision—measuring one characteristic of a particle with precision means that other related characteristics can be measured very imprecisely. The Uncertainty Principle reinforced the idea that nature behaves randomly, not deterministically as both Newton and Einstein believed.

As quantum theory advanced its predictions about tiny particles became increasingly strange and mysterious. But experimental evidence has always supported the theory: every discovery that quantum theory is wrong has itself been proven wrong. The theory works, but nobody understands why. The Complementarity Debate between

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Bohr and Einstein revealed that every proposed paradox could be explained by quantum theory, but the confusion about what quantum theory really means continues.

That confusion led to efforts to develop a quantum canon—an official interpretation of what quantum theory was, and of how it should be understood. Bohr and Heisenberg were instrumental in developing the Copenhagen Interpretation that documented the tenets of quantum theory, but they could not explain why those tenets are valid. The Copenhagen Interpretation is the "consensus" interpretation of quantum theory today, but it has rivals. Among them is the Parallel Universe explanation that explains the collapse of the wave function, but only at the expense of increased mystery.

In this first part of our exploration of quantum mechanics we have reviewed the history of its development, and we have looked at some experiments that reveal strange behavior at the subatomic level. In the second part of this trip through quantum mechanics we will address the theory itself, in the process explaining some of the mysteries that we have uncovered in our summary of the foundations.

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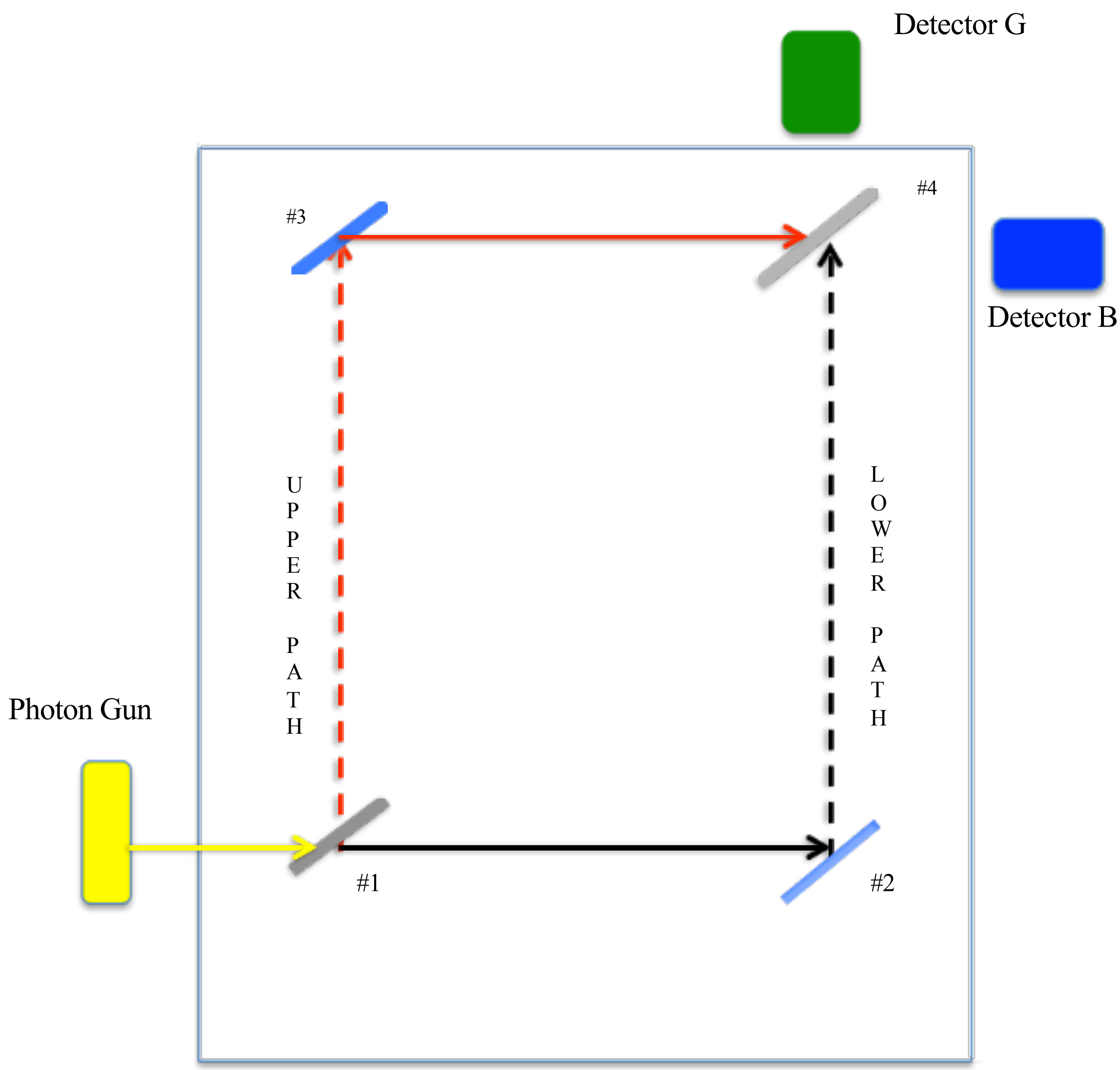
Appendix:

Interferometer Experiments

Figure 1

A Mach-Zehnder Interferometer



Legend

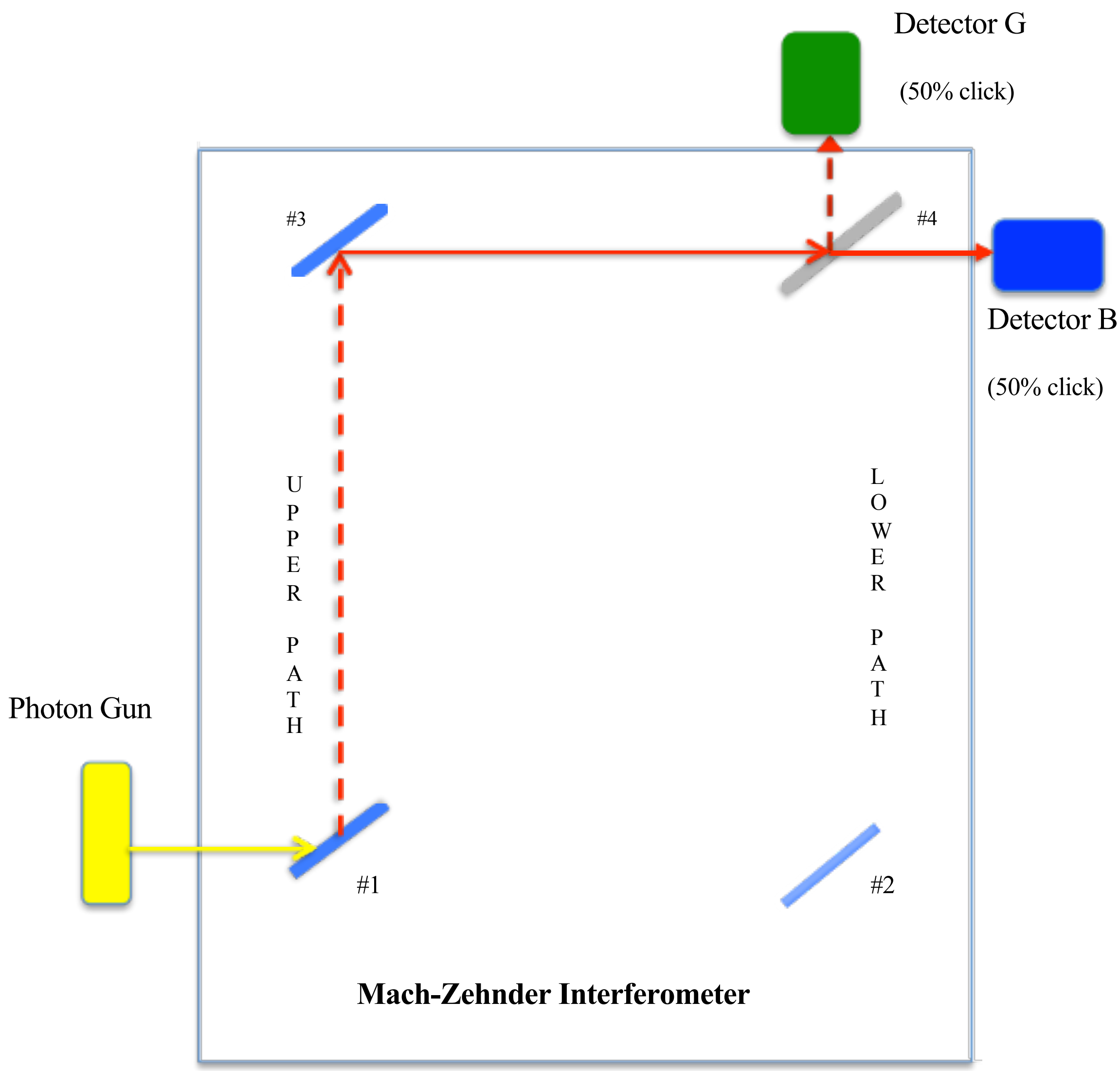
Red Line = Upper Path
Solid Line = Normal Wave Form
Unsilvered Mirror = Blue Bar

Black Line =Lower Path
Dashed Line = Inverted Wave Form
Half-Silvered Mirror = Silver Bar

Half-Silvered Mirrors are Glass on the North Side, Silvered on South Side

Figure 2

A One-Path Experiment



Legend

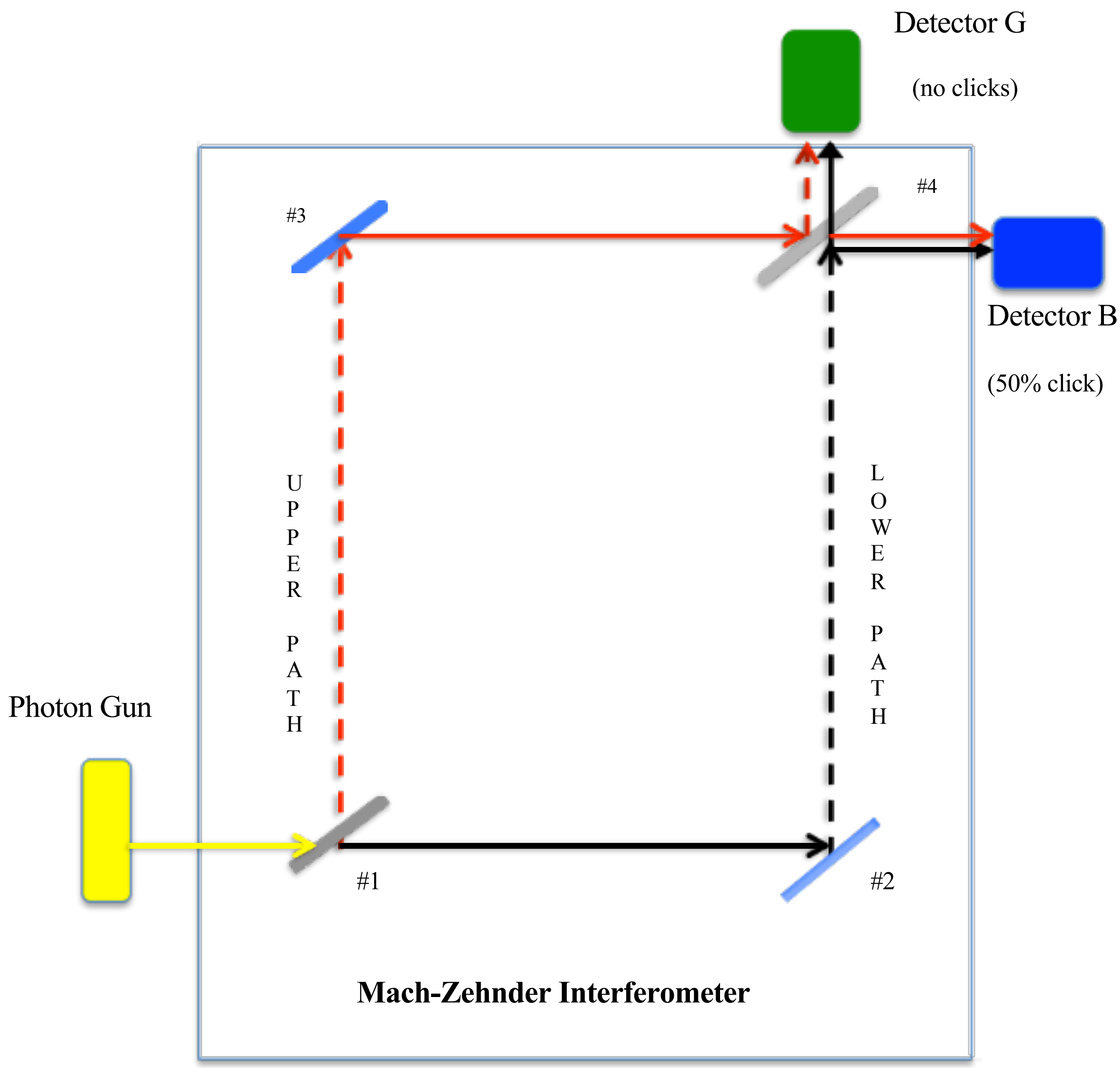
Red Line = Upper Path
Solid Line = Normal Wave Form
Unsilvered Mirror = Blue Bar

Black Line =Lower Path
Dashed Line = Inverted Wave Form
Half-Silvered Mirror = Silver Bar

Half-Silvered Mirrors are Glass on the North Side, Silvered on South Side

Figure 3

A Two-Path Experiment



Legend

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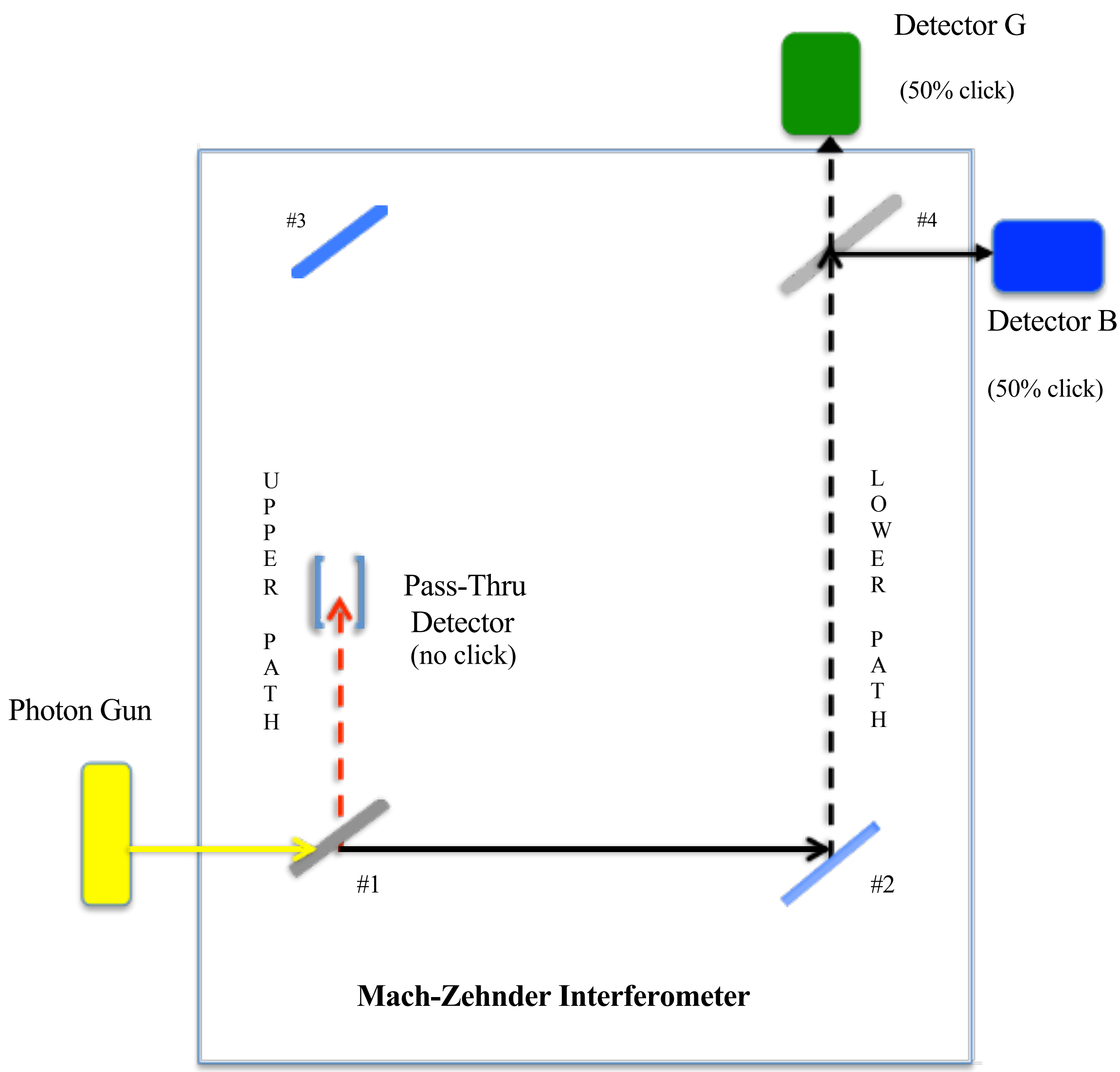
Solid Line = Normal Wave Form
Unsilvered Mirror = Blue Bar

Black Line =Lower Path
Dashed Line = Inverted Wave Form
Half-Silvered Mirror = Silver Bar

Half-Silvered Mirrors are Glass on the North Side, Silvered on South Side

Figure 4

A Which-Path Experiment



Legend

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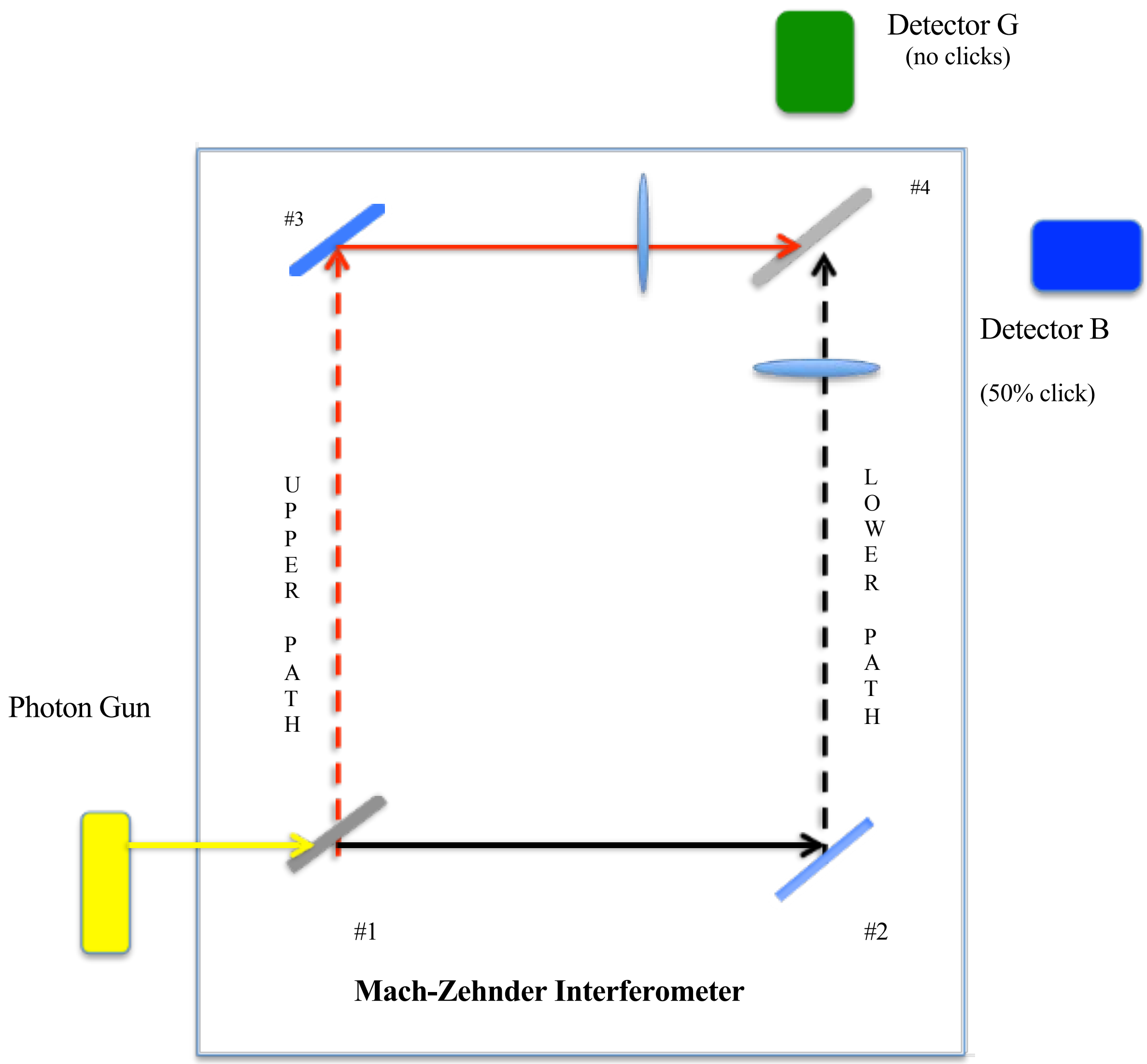
Black Line =Lower Path
Dashed Line = Inverted Wave Form
Half-Silvered Mirror = Silver Bar

Half-Silvered Mirrors are Glass on the North Side, Silvered on South Side

Figure 5

A Delayed-Choice Which-Path Experiment



Legend

Red Line = Upper Path
Solid Line = Normal Wave Form
Unsilvered Mirror = Blue Bar

Black Line =Lower Path
Dashed Line = Inverted Wave Form
Half-Silvered Mirror = Silver Bar

Half-Silvered Mirrors are Glass on the North Side, Silvered on South Side